

An Introduction to Paraconsistent Logic and its Applications

Can Başkent

ESSLLI 2018, University of Sofia, Bulgaria

1 Abstract

Paraconsistent logics are formal systems where contradictions do not entail everything. In paraconsistent logics, it is possible to have non-trivial inconsistent theories. This course introduces various well-known paraconsistent logics with an application oriented perspective. The applications are chosen from foundational issues in logic, mathematics and game theory with an aim of relating the subject to a wider audience with different backgrounds and interests. For that reason, the course targets an inter-disciplinary audience and serves as an introduction for those who are interested in non-classical logic, non-standard mathematics, set theory and game theory.

The primary objective of this course is to introduce some well-studied paraconsistent logics from an application oriented point of view. While achieving this, a delicate balance between the theory and the applications will be maintained. At the end of this course, students will learn more on the dependency of (game theoretical and mathematical) theories on logic. They will see how naïve theories can be revived by adopting an alternative logic and how logical methods can help build a non-classical theory. Consequently, this course will exhibit the use of paraconsistent logics *in action*. By adopting a *pluralistic* approach, paraconsistency will be introduced as a rich tool-kit for formal sciences with a lot to offer. Therefore, this course relates naturally to students who are interested in foundational studies, computational sciences and game theory as well as non-classical logic itself.

2 Tentative Outline

A brief and tentative outline of the course is summarized as follows.

Lecture 1 Introduction to Paraconsistent Logics. Logic of Paradox. First-Degree Entailment. Relevant Logic. An application: Translation between Logic of Paradox and Classical S5.

Lecture 2 Da Costa Systems. Logics of Formal Inconsistency. Preservationism. Logics of Nonsense.

Lecture 3 Game Semantics and Topological Semantics for Paraconsistent Logics. Dynamic Epistemic Logic in Paraconsistent Topologies. Homotopies.

Lecture 4 Applications in Foundations: Paraconsistent Set Theory. Paraconsistent Analysis. Paraconsistent topology.

Lecture 5 Applications in Games: Paradoxes of Games (a self-referential one and a non-self-referential one) and Paraconsistent Models for them. Review of the Course and open problems.

References

ANDERSON, A. R., & BELNAP, NUEL D. 1963. First Degree Entailments. *Mathematische Annalen*, **149**, 302–319.

ANDERSON, A. R., BELNAP, NUEL D., & DUNN, J. MICHAEL. 1992. *Entailment*. Princeton University Press.

- AVRON, ARNON, & ZAMANSKY, ANNA. 2011. Non-Deterministic Semantics for Logical Systems. *Pages 227–304 of: GABBAY, DOV M., & GUENTHNER, F. (eds), Handbook of Philosophical Logic*, vol. 16. Springer.
- BAŞKENT, CAN. 2013. Some Topological Properties of Paraconsistent Models. *Synthese*, **190**(18), 4023–4040.
- BAŞKENT, CAN. 2016. Game Theoretical Semantics for Some Non-Classical Logics. *Journal of Applied Non-Classical Logics*, **26**(3), pp. 208–39.
- BAŞKENT, CAN. 2018. A Yabloesque Paradox in Epistemic Game Theory. *Synthese*, **195**(1), 441–464.
- BEALL, JC, & RESTALL, GREG. 2006. *Logical Pluralism*. Clarendon Press.
- BRANDENBURGER, ADAM, & KEISLER, H. JEROME. 2006. An Impossibility Theorem on Beliefs in Games. *Studia Logica*, **84**, 211–240.
- CARNIELLI, WALTER A., & CONIGLIO, MARCELO E. 2016. *Paraconsistent Logic: Consistency, Contradiction and Negation*. Springer.
- CARNIELLI, WALTER A., CONIGLIO, M. E., & MARCOS, J. 2007. Logics of formal inconsistency. *Pages 15–107 of: GABBAY, DOV, & GUENTHNER, F. (eds), Handbook of Philosophical Logic*, vol. 14. Springer.
- DA COSTA, NEWTON C. A., KRAUSE, DÉCIO, & BUENO, OTÁVIO. 2007. Paraconsistent Logics and Paraconsistency. *Pages 655–781 of: JACQUETTE, D. (ed), Philosophy of Logic*, vol. 5. Elsevier.
- KOOI, BARTELD, & TAMMINGA, ALLARD. 2013. Three-valued Logics in Modal Logic. *Studia Logica*, **101**(5), 1061–1072.
- MORTENSEN, CHRIS. 1994. *Inconsistent Mathematics*. Springer.
- MORTENSEN, CHRIS. 2000. Topological Separation Principles and Logical Theories. *Synthese*, **125**(1-2), 169–178.
- MORTENSEN, CHRIS. 2010. *Inconsistent Geometry*. College Publications.
- PACUIT, ERIC. 2007. Understanding the Brandenburger-Keisler Belief Paradox. *Studia Logica*, **86**(3), 435–454.
- PRIEST, GRAHAM. 2002. Paraconsistent Logic. *Pages 287–393 of: GABBAY, DOV, & GUENTHNER, F. (eds), Handbook of Philosophical Logic*, vol. 6. Kluwer.
- PRIEST, GRAHAM. 2006. *In Contradiction*. 2. edn. Oxford University Press.
- PRIEST, GRAHAM. 2008. *An Introduction to Non-Classical Logic*. Cambridge University Press.
- RAHMAN, SHAHID, & TULENHEIMO, TERO. 2009. From Games to Dialogues and Back. *Pages 153–208 of: MAHER, ONDREJ, PIETARINEN, AHTI, & TULENHEIMO, TERO (eds), Games: Unifying Logic, Language and Philosophy*. Springer.
- SCHOTCH, PETER, BROWN, BRYSON, & JENNINGS, RAYMOND (eds). 2009. *On Preserving: Essays on Preservationism and Paraconsistent Logic*. University of Toronto Press.
- WEBER, ZACH, & COTNOIR, A. J. 2015. Inconsistent Boundaries. *Synthese*.
- WEBER, ZACH, & MCKUBRE-JORDENS, MAARTEN. 2012. Real Analysis in Paraconsistent Logic. *Journal of Philosophical Logic*, **41**(5), 901–922.