An Epistemic - Geometric Extension of Game Logic

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Contents

How to Start?

What about the games with uncertain moves?

Consider the dart game: you aim at a point, and the dart hits at a point *around* your aim. By construction, there is *some* uncertainty involved. Assuming the players are rational, you can assume *some* level of uncertainty as they will not aim at somewhere other than the dart board.

Thus, notion of *closure* which is conceptually familiar from topology can be used to understand uncertainty in dynamic situations.

Road Map

We will consider two well-defined logics: an epistemic one and a dynamic/game theoretical one. Then merge them in a meaningful way.

Epistemic constructions will then emphasize the strategies and will make them the focus of our work[1].

[1]Thanks to R.Ramanujam for pointing this out.

Hidden Agenda

We will utilize a dynamic logic which depends on Propositional Dynamic Logic. Thus, our game theoretical approach is a step towards the geometrical understanding of dynamic logics (one-sorted or many-sorted).

Basics

Subset space logic (SSL) formalizes reasoning about sets and points with an underlying motivation of embedding the geometrical notion of *closeness* into epistemic logic [3].

The key idea of SSL can be formulized as follows: "In order to *get close*, one needs to spend some *effort*." Thus, In SSL, the knowledge is defined with respect to both a *point* and a *neighborhood* of that point.

A subset space model is a triple $\langle S, \sigma, v \rangle$ where S is a set of points and $\sigma \subseteq \wp(S)$ and v is a valuation function.

Syntax and Semantics

We have two modalities: Knowledge (K) and Effort ($\Box$) with the usual syntax.

$$
\begin{array}{lll}
s, U \models p & \text{iff} & s \in v(p) \\
s, U \models \varphi \wedge \psi & \text{iff} & s, U \models \varphi \text{ and } s, U \models \psi \\
s, U \models \neg\varphi & \text{iff} & s, U \not\models \varphi \\
s, U \models \mathrm{K}\varphi & \text{iff} & t, U \models \varphi \text{ for all } t \in U \\
s, U \models \Box\varphi & \text{iff} & s, V \models \varphi \text{ for all } V \subseteq U \text{ for } V \in \sigma
\end{array}
$$

Axioms

The axioms of SSL simply reflect the fact that the K modality is S5-like whereas the $\Box$ modality is S4-like. Moreover, we need an additional axiom to state the interaction between the two modalities: $\mathsf{K}\Box\varphi \rightarrow \Box\mathsf{K}\varphi$. Yet another important fact is that the atomic sentences are independent from their neighborhoods, thus the following axiom for atomic sentence F is valid in SSL: $(F \rightarrow \Box F) \wedge (\neg F \rightarrow \Box\neg F)$. Moreover, SSL is sound and complete with respect to the aforementioned axiomatization. Furthermore, it is decidable.

Basics

Game logic (GL) uses the constructive ideas which are familiar from PDL in order to give an abstract framework for games [2, 4]. The games in GL have two players which we call ∃loise and ∀belard. In order to be able to construct the set of well-formed formulae of GL, we need a set of atomic propositions Π and a set of atomic games Γ.

Syntax

Syntax of GL is as follows.

$$\gamma := g \mid \varphi? \mid \gamma;\gamma \mid \gamma \cup \gamma \mid \gamma^* \mid \gamma^d$$
$$\varphi := \bot \mid p \mid \neg\varphi \mid \varphi \vee \varphi \mid \langle\gamma\rangle\varphi$$

A Model for Games

A model $\mathcal{M}$ of GL is the triple $\mathcal{M} = \langle S, \{E_g : g \in \Gamma\}, V \rangle$ where S is a set of states, V is a valuation function, and a family of effectivity functions $E_g : S \to \wp(\wp(S))$ which are monotonic [4].

In other words, our models here are neighborhood models.

Semantics

Since Boolean cases are as usual, we skip them and give the semantics of the modal operator here.

$$\mathcal{M}, s \models \langle\gamma\rangle\varphi \text{ iff } (\varphi)^{\mathcal{M}} \in E_\gamma(s)$$

Why?

An important deficiency of GL is the fact that it does not address the epistemic aspects of the games. Our goal in this work is to offer an extension of GL in order to be able supplement GL with the aforementioned missing component and equip it with a geometrical semantics as the geometrical semantics is the natural candidate for reasoning about closeness and approximation.

Extended Syntax

$$\gamma := g \mid \varphi? \mid \gamma;\gamma \mid \gamma \cup \gamma \mid \gamma^* \mid \gamma^d$$
$$\varphi := \mid p \mid \neg\varphi \mid \varphi \vee \varphi \mid \mathsf{K}_\gamma\varphi \mid \Box_\gamma\varphi \mid \langle\gamma\rangle\varphi$$

Semantics

$\mathcal{M} = \langle S, \{\tau_\gamma^{s,i} : \gamma \in \Gamma, s \in S, i \in \mathrm{A}\}, V\rangle$ where S is a set, V is a valuation, the family $\{\tau_\gamma^{s,i}\}$ is a set of subsets of S (i.e. strategies) associated with the agent i at the state s for the game γ.

$$
\begin{array}{lll}
s, U \models p & \textit{iff} & s \in V(p) \\
s, U \models \varphi \wedge \psi & \textit{iff} & s, U \models \varphi \textit{ and } s, U \models \psi \\
s, U \models \neg\varphi & \textit{iff} & s, U \not\models \varphi \\
s, U \models \mathrm{K}_\gamma \varphi & \textit{iff} & t, U \models \varphi \textit{ for all } t \in U \in \tau_\gamma^{s,i} \\
s, U \models \Box_\gamma \varphi & \textit{iff} & s, V \models \varphi \textit{ for all } V \subseteq U \textit{ for } V \in \tau_\gamma^{s,i} \\
s, U \models \langle\gamma\rangle\varphi & \textit{iff} & (s, U) \in (\varphi)^{\mathcal{M}} \textit{ for } s \in U \in \tau_\gamma^{t,i}
\end{array}
$$

Axioms

We will adopt the S5 axiomatization for the epistemic modality and S4 axiomatization for the effort modality. The axiomatization of EGL follows the intuition behind the basic game logic.

- $\langle \gamma \cup \delta \rangle \varphi \leftrightarrow \langle \gamma \rangle \varphi \vee \langle \delta \rangle \varphi$
- $\langle \gamma ; \delta \rangle \varphi \leftrightarrow \langle \gamma \rangle \langle \delta \rangle \varphi$
- $\langle \psi ? \rangle \varphi \leftrightarrow (\psi \wedge \varphi)$
- $(\varphi \vee \langle \gamma \rangle \langle \gamma^* \rangle \varphi) \leftrightarrow \langle \gamma^* \rangle \varphi$
- $\langle \gamma^d \rangle \leftrightarrow \neg \langle \gamma \rangle \neg \varphi$

and

- $\mathsf{K}_\gamma \Box_\gamma \varphi \rightarrow \Box_\gamma \mathsf{K}_\gamma \varphi$
- $\mathsf{L}_\gamma \langle \gamma \rangle \varphi \leftrightarrow \langle \gamma \rangle \mathsf{L}_\gamma \varphi$
- $\Diamond_\gamma \langle \gamma \rangle \varphi \leftrightarrow \langle \gamma \rangle \Diamond_\gamma \varphi$

Strategy Based Interpretation

Strategies specifies *how/where* we know the information.

Epistemically, it addresses where we can know the information in question (`go to point` x in the neighborhood U).

Dynamically, it addresses how we can reach this knowledge situation (`Shrink/Improve your information to the subset` V `at` x).

Research Directions

Further Work

- **Completeness** of Game Logic is still unproven.
- **Geometrical Semantics** for Dynamic Logics
- **Uncertainty** in games discussed with the idea of closeness/neighborhoods

Some References

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Thanks!

Talk slides and the preliminary report is available at:

www.canbaskent.net