What is Game Theoretical Negation?

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What is Hintikka's Game Theoretical Semantics? I

The *semantic verification game* is played by two players, traditionally called Abelard (after $\forall$) and Eloise (after $\exists$), and the rules are specified syntactically.

During the game, the given formula is broken into subformulas by the players step by step, and the game terminates when it reaches the propositional atoms.

If we end up with a propositional atom which is true in the model in question, then Eloise wins the game. Otherwise, Abelard wins. We associate conjunction with Abelard, disjunction with Heloise.

The major result of this approach states that Eloise has a winning strategy if and only if the given formula is true in the model.

What is Hintikka's Game Theoretical Semantics? II

When conjunction and disjunction are considered, game theoretical semantics is very appealing. In negated formulas, game theoretical semantics says that the players switch their roles. Abelard takes up Eloise's verifier role, and Eloise becomes the falsifier.

I think this is counter-intuitive - game theoretically.

An Example I

An Example

Two men want to marry a princess. The king says they have to race on a horceback. The slowest one wins, and can marry the princess. How can one win this game and marry the princess?

The answer: men need to swap their horses. Since the fastest lose, and players race with each other's horses, what they need to do is to become the fastest in the dual game. Fastest one in the switched horse, considered as the negation of the slowest in the dual game, wins the game.

An Example II

In this example, GTS for negation becomes evident. If the slowest one wins the game, then the fastest one wins the dual game.

There is certainly some sense of rationality here. Namely, the players consider it easier to switch horses and race in the dual game.

Namely, can we play chess in this way? Can we play football in this fashion? Is it always rational to play in the dual game with switched roles?

To switch to the easier dual game with switched roles is a **meta-game theoretical** move. This is not a strategy within the given game, it is a *strategy on the games and over the games*.

Non-classical Games

It is not difficult to introduce additional outcomes for GTS. We introduce the following five non-classical / non-zero sum possibilities:

1. Abelard and Eloise both win.
2. Abelard and Eloise both lose.
3. Eloise wins, Abelard does not lose.
4. Abelard wins, Eloise does not lose.
5. There is a tie.

What are the Non-classical Games?

Some propositions can belong to both player: namely, both the proposition and its negation can be true.

Some propositions can belong to the neither: namely, neither the proposition nor its negation can be true.

Some propositions may not belong to one player without the negation belonging to the opponent: namely, the proposition can be true, but its negation may not be false.

In short, the game does not have to be a zero-sum game. One's win may not imply the other's loss.

Logic of Paradox and GTS

The formalism we adopt here is Graham Priest's Logic of Paradox (Priest, 1979). The logic of paradox (LP, for short) introduces an additional truth value P, called *paradoxical*, that stands for both true and false.

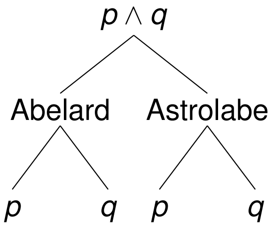
	$\neg$
T	F
P	P
F	T

$\wedge$	T	P	F
T	T	P	F
P	P	P	F
F	F	F	F

$\vee$	T	P	F
T	T	T	T
P	T	P	P
F	T	P	F

Game Theoretical Semantics for LP

Consider the conjunction again. Take the formula $p \wedge q$ where p, q are P, F respectively.



Abelard makes a move, and as the falsifier, he chooses q which is false. This gives him a win. Interesting enough, Astrolabe also makes a move and, chooses p giving him a win. In this case both have a winning strategy. Moreover, the win for Abelard does not automatically entail that it is a loss for Astrolabe.

Game Rules for LP

Denote it with GTS^{pp}.

p	whoever has p in their extension, wins
$F \wedge G$	Abelard and Astrolabe chooses between F and G simultaneously
$F \vee G$	Eloise and Astrolabe chooses between F and G simultaneously
$\neg(F \wedge G)$	Eloise and Astrolabe chooses between $\neg F$ and $\neg G$ simultaneously
$\neg(F \vee G)$	Abelard and Astrolabe chooses between $\neg F$ and $\neg G$ simultaneously

Correctness

Theorem

In GTS^{pp} verification game for φ,

- Eloise has a winning strategy if φ is true
- Abelard has a winning strategy if φ is false
- Astrolabe has a winning strategy if φ is paradoxical

Correctness

Theorem

In a GTS^{pp} game for a formula φ in a LP model M,

- If Eloise has a winning strategy, but Astrolabe does not, then φ is true (and only true) in M
- If Abelard has a winning strategy, but Astrolabe does not, then φ is false (and only false) in M
- If Astrolabe has a winning strategy, then φ is paradoxical in in M

Different Truths and Different Wins

Paraconsistency distinguishes different *true*s:

- Trues that are only true
- Trues that are also false

(and similarly for the *false*s)

The game semantics for it also distinguishes different *win*s

- Wins that are my wins and your loss
- Wins that are only my wins (not necessarily your loss)

(and similarly for the *loss*es)

Dominating Strategies

Note that in the weaker version, we simply eliminate the dominated strategies (by embedding the players' rationality in the semantics), and iterate the procedure.

Thus, it can be seen as an iterated elimination of dominated strategies - which is not visible in the classical case, but clearer in the paraconsistent case - due to the truth table of LP.

LP truth table

Conclusion I

In this work, we do not aim at giving a full picture of game theoretical semantics of negation in all non-classical logics. The literature on non-classical logics (which include intuitionistic, paraconsistent and relative logics amongst many others) is vast, and all of those logics are not transformable to each other making it almost impossible to give a unifying theme for GTS.

Yet, the very same intuition can easily be applied to other non-classical logics, and their winning conditions can be examined.

Conclusion II

In a recent paper, Priest alludes to similar concepts (Priest, 2013). We can add some further points by noting that our approach here can be a case for the plurality of logic.

Similarly, Dialogical Logic can initially be taught of providing a good approach to negation. However, a closer inspection reveals that in dialogical logical cases, the role switching idea is maintained and even taken to a higher level creating more schizophrenic players (Rahman & Tulenheimo, 2009).

Conclusion III

Behavioral economics and the charming examples that it provides (for example (Ariely, 2008; Ariely, 2010; Harford, 2009)) constitutes an interesting playground for the ideas we have developed here.

Thanks for your attention!

Talk slides and the papers are available at

`www.CanBaskent.net/Logic`

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