A Paraconsistent Logic for Contrary-to-Duty Obligations

Can BAŞKENT

Department of Computer Science, University of Bath, England

can@canbaskent.net www.canbaskent.net/logic

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Outlook of the Talk

- Introduction
- Paraconsistent Formalism
- Dynamic Paraconsistent Formalism
- Applications and Conclusion

Contrary to Duty Obligations

Moral philosopher Roderick Chisholm defines contrary to duty (CtD) obligations as the “Exhortations [which] often take the form: ‘You ought to do *a*, but if you do not do *a*, then you must by all means, do *b*’ ”. Therefore, contrary to duty obligations are those that tell us “what we ought to do if we neglect certain of our duties” (Chisholm, 1963).

Paraconsistency

CtD obligations can be formalized in a variety of ways. We suggest a broader framework that underlines the **inconsistent** nature of CtD obligations and deontologies.

A logic is **paraconsistent** if contradictions do not trivialize it. In paraconsistent logics, there can exists propositions that are both true and false.

The paraconsistent logic we are concerned with in this paper has been developed by da Costa and his colleagues, and is one of the well-studied systems of paraconsistency (da Costa, 1974; da Costa & Alves, 1977).

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The System C_1^D

Da Costa and Carnielli suggested a paraconsistent deontic logic C_1^D based on da Costa's well studied paraconsistent logic C_1 (da Costa & Carnielli, 1986).

The propositional system C_1^D is constructed with the standard propositional syntax given a denumerably infinite set of propositional variables **P**, and it admits a negation operator $\neg$ and a conjunction operator $\wedge$.

C_1^D distinguishes two kinds of propositions: the **good** ones that are classical and satisfy the law of contradiction, and the **bad** ones that are not classical and do not satisfy the law of contradiction.

Good propositions satisfy $\varphi^\circ := \neg(\varphi \wedge \neg\varphi)$.

The System C_1^D

The basic system admits the following axiom schemes (da Costa & Alves, 1977).

- $\varphi \vee \neg\varphi$
- $\neg\neg\varphi \to \varphi$
- $\varphi \to (\psi \to \varphi)$
- $\varphi \wedge \psi \to \varphi$
- $\varphi \wedge \psi \to \psi$
- $\varphi \to (\psi \to \varphi \wedge \psi)$
- $\varphi \to \varphi \vee \psi$
- $\varphi \to \psi \vee \varphi$
- $(\varphi \to \psi) \to ((\varphi \to (\psi \to \chi)) \to (\varphi \to \chi))$
- $(\varphi \to \chi) \to ((\psi \to \chi) \to (\varphi \vee \psi \to \chi))$
- $\psi^\circ \to ((\varphi \to \psi) \to ((\varphi \to \neg\psi) \to \neg\varphi))$
- $\varphi^\circ \wedge \psi^\circ \to ((\varphi \to \psi)^\circ \wedge (\varphi \wedge \psi)^\circ \wedge (\varphi \vee \psi)^\circ)$

The rule of inference in C_1 is modus ponens.

Some Invalidities

The following list includes some of the *invalidities* in C_1 which maybe helpful (da Costa & Alves, 1977).

- $\varphi \wedge \neg\varphi \rightarrow \psi$
- $\varphi \wedge \neg\varphi \rightarrow \neg\psi$
- $\neg(\varphi \wedge \neg\varphi)$
- $\varphi \rightarrow (\neg\varphi \rightarrow \psi)$
- $\neg\varphi \rightarrow (\varphi \rightarrow \psi)$
- $\varphi \equiv \neg\neg\varphi$
- $\neg(\varphi \vee \psi) \equiv \neg\varphi \wedge \neg\psi$
- $\neg(\varphi \wedge \psi) \equiv \neg\varphi \vee \neg\psi$

Deontic System: Syntax

The syntax of C_1^D introduces a unary modal operator O that stands for "it is obligatory that", and the formulas are defined in the usual way closing them under the standard connectives of C_1 and the modal operator O.

The additional axioms of C_1^D are given as follows.

- $O(\varphi \to \psi) \to (O\varphi \to O\psi)$
- $O\varphi \to \sim O \sim \varphi$
- $\varphi^\circ \to (O\varphi)^\circ$
- $\vdash \varphi \therefore \ \vdash O\varphi$

Some Validities

The following is *valid* in C_1^D.

- $O\varphi \to O(\varphi \vee \psi)$
- $O(\varphi \wedge \psi) \equiv O\varphi \wedge O\psi$
- $\sim(O\varphi \wedge \sim O\varphi)$
- $O\varphi \wedge O(\varphi \to \psi) \to O\psi$

Some Invalidities

The following is *invalid* in C_1^D.

- $O\neg(\varphi \wedge \neg\varphi)$
- $O(\varphi \wedge \neg\varphi) \to O\psi$
- $O\varphi \wedge O\neg\varphi \to O\psi$
- $\neg(F\varphi \wedge P\varphi)$

Deontic System: Modal Semantics

The model is a standard one: $M = (W, R, V)$ where W is a non-empty set, R is a serial binary accessibility relation defined on W, and V is a valuation function. The semantics for propositional variables, conjunction, disjunction and the implication are standard. For the rest, it is given as follows for $w \in W$ (da Costa & Carnielli, 1986).

- $w \models \neg\varphi$ when $w \not\models \varphi$,
- $w \models O\varphi$ iff for all $w' \in W$ such that wRw', $w' \models \varphi$,

Based on this semantics and axiomatization, da Costa and Carnielli showed that C_1^D is sound, complete and decidable.

Contrary to Duty Obligations in Modal Syntax

We introduce the following dyadic modality $C(\varphi, \psi)$ for well-formed formulas φ, ψ and call this system C_1^{DD}.

The expression $C(\varphi, \psi)$ reads "it is obligatory that φ, yet if φ is not the case, then it is obligatory that ψ" with the following semantics.

$$M, w \models C(\varphi, \psi) \quad \textit{iff} \quad M, w \models O\varphi \wedge \neg\varphi \to O\psi$$

Now, we can have satisfiable inconsistencies: take ψ as $\neg\varphi$. Then, $C(\varphi, \neg\varphi)$ reduces to $O\varphi \wedge \neg\varphi \to O\neg\varphi$. If $O\varphi \wedge \neg\varphi$ holds, then we have $O(\varphi \wedge \neg\varphi)$ which is contradictory.

Contrary to Duty Obligations: Some Observations I

Proposition

The statement $\neg C(\varphi, \neg\varphi)$ is not valid in C_1^{DD}.

Theorem

The logic C_1^{DD} is complete and decidable with respect to the given axiomatization of C_1^D.

Contrary to Duty Obligations: Some Observations II

In C_1^{DD}, inconsistent obligations do not necessarily generate absurdities.

Proposition

In C_1^{DD}, $C(\varphi, \neg\varphi) \to \psi$ and $C(\varphi \wedge \neg\varphi, \psi)$ are not valid. However, $\neg C(\varphi^\circ, \neg\varphi^\circ)$ is valid.

Contrary to Duty Obligations: Some Observations III

We can translate the modal definition of CtD to first-order language by the Sahlqvist Algorithm (Blackburn *et al.*, 2001) to obtain $\forall y \forall z (Rxy \wedge x \neq y \rightarrow Rxz)$.

Proposition

Contrary to Duty modality $C(\cdot, \cdot)$ is definable in the first-order language.

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Dynamic Deontic Paraconsistent Logic for Contrary to Duty Obligations

Given a C_1^{DD} model $M = (W, R, V)$, we define *updated model* $M|\varphi = (W', R', V')$ as such.

We first set $[\varphi]^M = \{w \in M : \forall v.wRv \to M, v \models \varphi\}$. In other words, $[\varphi]^M$ is the possibly empty set of states at which φ is obligatory. Then, $W' := [\varphi]^M$, $R' := R \cap (W' \times W')$, and similarly $V' := V \cap W'$.

Dynamic Contrary to Duty Obligations: Definition

Now we give a dynamic semantics for $C(\cdot, \cdot)$ as follows.

$$M, w \models C(\varphi, \psi) \text{ iff } M, w \models \neg\varphi \text{ implies } M|\varphi, w \models O\psi$$

This approach makes it clear how a *violation* can be viewed as a dynamic and deontic *update*.

Similarly, in public announcement logic, an epistemic announcement becomes known after it is announced. In $M|\varphi$, the formula φ becomes obligatory after the model is updated accordingly.

Dynamic Contrary to Duty Obligations: Some Observations I

Proposition

It is not necessarily the case that $[\varphi]^M \cap [\neg\varphi]^M = \emptyset$.

Proposition

$[\varphi^\circ]^M \cap [\sim\varphi^\circ]^M = \emptyset$.

Dynamic Contrary to Duty Obligations: Some Observations II

Proposition

$M|\varphi, w \models O\varphi$ for any φ.

Proposition

$M, w \models O\varphi \rightarrow \psi$ if $M|\varphi, w \models \psi$ for any φ, ψ.

Dynamic Contrary to Duty Obligations: Some Observations III

Proposition

$M|\varphi, w \models O\neg\varphi$ is satisfiable for some M and w.

Proposition

$\not\models \varphi^\circ \rightarrow C(\varphi^\circ, \neg\varphi^\circ)$.

Dynamic Contrary to Duty Obligations: Some Observations IV

Proposition

$M|\varphi|\psi = M|(\varphi \wedge \psi)$.

Proposition

$M|\varphi^\circ|\neg\varphi^\circ = \emptyset$.

Dynamic Contrary to Duty Obligations: Some Observations V

Proposition

The domain of $M|\varphi|\neg\varphi$ (or $M|\varphi \wedge \neg\varphi$) is not necessarily the empty set in C_1^{DD}.

Proposition

$M|\varphi \models O\psi$ iff $M|\varphi|\psi = M|\varphi$, for any model M and formulas φ, ψ.

Dynamic Contrary to Duty Obligations: Some Observations VI

Theorem

For any model M and any formula φ, $M \models O\varphi$ if and only if M has a fixed-point for dynamic deontic updates at φ, i.e $M|\varphi = M$.

Theorem

$M, w \models C(\varphi, \psi)$ if $M|\varphi|\psi, w \models \neg\varphi$, for any model M and formulas φ, ψ.

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Conditional Obligations

Van Fraassen defines a conditional obligation operator $O(\varphi, \psi)$ which reads as "under conditions satisfying ψ, φ ought to be satisfied" (van Fraassen, 1972). In this reformulation, the unary modality $O(\varphi)$ is defined as $O(\varphi, \psi \to \psi)$.

A connection between CtD obligations and conditional obligations was made in (Tomberlin, 1981), now we will make this connection dynamic and inconsistency-friendly.

Conditional vs Contrary to Duty Obligations

CtD obligations can be represented by conditional obligations:
$C(\varphi, \psi) := O(\psi, \neg\varphi \wedge O(\varphi, \psi \rightarrow \psi))$

or equivalently $C(\varphi, \psi) := O(\psi, \neg\varphi \wedge O\varphi)$.

Some theorems of conditional obligations hold in C_1^{DD}:
$O(\varphi, \psi) \equiv O(\varphi \vee \neg\psi, \psi)$.

Inconsistent Conditional Obligations

Proposition

For any model M, and a state w in it, $M|\varphi, w \models \psi$ implies $M|\varphi, w \models O(\varphi, \psi)$ for any formulas φ, ψ.

The logic C_1^{DD} allows some relatively counter-intuitive statements including $O(\varphi, O\neg\varphi)$.

Computer Systems Security

The formula $C^{\circ}(\varphi, \psi)$ can be viewed as a security restriction which suggests that $O\varphi$ and $O\psi$ can never be contradictory. Identifying "good" formulas with security restrictions and constraints that can never be breached, and "bad" formulas with the propositions whose contradictions can be *tolerated* is a fruitful approach in systems' security. Simply put, it is forbidden to jaywalk, yet *usually* its violations are tolerated. However, it is also forbidden to murder people, but its violations are *rarely* tolerated.

Philosophy of Medicine

According to Sadegh-Zadeh, "... there are two additional problems that reduce the applicability of classical logic in medicine. The first one is the inconsistency of medical knowledge and the data" (Sadegh-Zadeh, 2011).

As Sadegh-Zadeh put it "... when set KB is the inconsistent knowledge base of the expert system and D is the set of data of an individual patient for whom a diagnosis is sought, the inference engine will infer from the inconsistent set $\mathrm{KB} \cup \mathrm{D}$ arbitrary statements about the patient, including false ones. [...] Such trivializations of knowledge bases can be prevented by using, instead of classical logic, a *paraconsistent* logic as the underlying logic of the inference engine" (Sadegh-Zadeh, 2012).

Diagnosis

French Paradox: Until recently, it was thought that dietary intake of cholesterol contributes heavily to blood cholesterol which in turn leads to cardio-vascular diseases. In line with this example, in general, French people are observed to intake a large amount of dietary cholesterol. Based on the knowledge base, it can be claimed that they "ought to have cardio-vascular diseases", or Od. Yet, the data shows that they indeed, perhaps paradoxically, do not, or $\neg d$. There suggested a variety of medical reasons, ranging from consumption of red wine during meals to having less stressed and slow meals, to account for this "paradox".

Diagnosis

Whatever the medical reason is for the French paradox, it is observed that what is entailed by the medical knowledge base and what is observed from the collected data are in conflict. Yet, under this contradictory information, there can be medically relevant advise to follow, say, to minimize the cardio-vascular risk, such as avoiding smoking and committing to an exercise regimen. In this case, if s denotes avoiding smoking, we have $Od \wedge \neg d \rightarrow Os$.

The dynamic approach to CtD obligations is useful in diagnostic reasoning as the dynamic update of the knowledge base does not *overkill* by imposing a hard elimination and removing the situations that can be helpful understanding the medical condition during diagnosis. So, it seems to align perfectly with what was alluded by Sadegh-Zadeh.

Conclusion

This paper establishes a bridge between the two research programs, dynamic modal agenda and parconsistent logics, by proposing a paraconsistent modal approach to contrary to duty obligations.

Thanks for your attention!

Talk slides and the paper are available at

`www.CanBaskent.net/Logic`

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