A Formal Epistemological Approach to Meno

yes, it is mathematical!

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Contents

Why Formalism is Important?

Emergence of Kripke semantics made it easy to work on modal logics in a formal setting.
Epistemic, doxastic, temporal and deontic modalities gained a lot of importance in computer science, relational model theory, philosophy, economics, game theory, linguistics and even in law.

What is Formalism?

Formalism is mathematics. Logic is mathematics.

Completeness

Here are the fundamental logical meta statements.

A system is sound if every provable statement is true. Soundness is relatively easy to establish.

A system is complete if every true statement is provable. It is difficult (See Henkin construction (i.e. adding constants) for the completeness of first-order logic).

Henkin's memoirs are sincere and honest on how he first came up with his proof.

Epistemic Logic

Epistemic logic is a modal logic which aims at a formalization of knowledge. It is a *formal epistemological* enterprise.

Notationwise, $K_i\varphi$ reads that *the agent i knows that* φ.

It has three important properties identified by Hintikka.

- $K_i\varphi \rightarrow \varphi$. Veridicality
- $K_i\varphi \rightarrow K_iK_i\varphi$. Positive Introspection
- $\neg K_i\varphi \rightarrow K_i\neg K_i\varphi$. Negative Introspection

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Various criticisms can be raised against each axiom. For example, people do not always possess positive or negative introspection. Socratic method, in this respect, tries to establish (positive or negative) introspection by *recalling*.

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Semantics

Topological (McKinsey and Tarski)

If our knowledge space forms a topology (T, τ), then we can give meaning to $\mathrm{K}\varphi$. Define a propositional valuation on the set S to get a model M.
For simplicity, we will consider the single agent case: only one knower.

Definition (Topological semantics for epistemic modality)

$M, w \models \mathrm{K}\varphi$ if and only if $\exists U \in \tau$ with $w \in U$ s.t. $\forall v \in U$, we have $M, v \models \varphi$.

This definition is the oldest semantics for modal logic: 1944.

Semantics

Graph Theoretical (Kripke)

We can use the possible world semantics that goes back to Leibniz (ontological argument: "god created the best of all possible worlds"). In modern era, it was first Carnap who tried and failed to present a precise semantics for modalities (1950s).
Assume, knowledge modality and its underlined relation forms a *graph* in the model M.

Definition (Kripke semantics for modal logic)

$M, w \models \mathrm{K}\varphi$ *if and only if* $\forall v(wRv \rightarrow M, v \models \varphi)$.

This definition is less complex (Π_1 compared to Σ_2) and more recent.

Interlude: Linguistics

Naming and Necessity

Modal approach is an attack to Russellian descriptive theory of proper names: *rigid designators*.

A very brilliant account since Frege, Russell, Wittgenstein.
Important discussions among Putnam, Searle, Kaplan, Devitt etc.

Necessity and possibility semantics is also given for modalities.

A similar problem: How to express "I came home and took a shower." in formal logic where "and" is commutative.

Formalism in Action!

I used to try to convince my girlfriend to read Meno after we had an argument.

Yes, it is Platonic.

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Our Focus

{Socrates is describing the square.}

S And if one side of the figure be of two feet, and the other side be of two feet, how much will the whole be? Let me explain: if in one direction the space was of two feet, and in other direction of one foot, the whole would be of two feet taken once?
B Yes.
S But since this side is also of two feet, there are twice two feet?
B There are.
S Then the square is of twice two feet?
B Yes.
S And how many are twice two feet? count and tell me.

Our Focus

B Four, Socrates.
S And might there not be another square twice as large as this, and having like this the lines equal?
B Yes.
S And of how many feet will that be?
B Of eight feet.
S And now try and tell me the length of the line which forms the side of that double square: this is two feet-what will that be?
B Clearly, Socrates, it will be double.
S Do you observe, Meno, that I am not teaching the boy anything, but only asking him questions; and now he fancies that he knows how long a line is necessary in order to produce a figure of eight square feet; does he not?

Our Focus

S Mark now the farther development. I shall only ask him, and not teach him, and he shall share the enquiry with me: and do you watch and see if you find me telling or explaining anything to him, instead of eliciting his opinion. Tell me, boy, is not this a square of four feet which I have drawn?
B Yes.
S And now I add another square equal to the former one?
B Yes.
S And a third, which is equal to either of them?
B Yes.
S Suppose that we fill up the vacant corner?
B Very good.

Our Focus

S Here, then, there are four equal spaces?
B Yes.
S And how many times larger is this space than this other?
B Four times.
S But it ought to have been twice only, as you will remember.
B True.
S And does not this line, reaching from corner to corner, bisect each of these spaces?
B Yes.

Our Focus

S And are there not here four equal lines which contain this space?
B There are.
S Look and see how much this space is.
B I do not understand.
S Has not each interior line cut off half of the four spaces?
B Yes.
S And how many spaces are there in this section?
B Four.
S And how many in this?
B Two.

Our Focus

S And four is how many times two?
B Twice.
S And this space is of how many feet?
B Of eight feet.
S And from what line do you get this figure?
B From this.
S That is, from the line which extends from corner to corner of the figure of four feet?
B Yes.
S And that is the line which the learned call the diagonal. And if this is the proper name, then you, Meno's slave, are prepared to affirm that the double space is the square of the diagonal?
B Certainly, Socrates.

Deduction?

What can we learn from a deduction?
Socrates' proof proceeded by state elimination. He discarded some previously thought possible worlds, and at each step approximated to the knowledge.
We make our topology finer and finer at each step.

Topology of Dynamic Models

Thus, from the topology $\langle T, \tau \rangle$, we obtain $\langle T, \tau' \rangle$ where τ' is a finer subtopology of τ. It is finer, because we learned something not by adding information, but reducing possibilities.

We can represent it by considering a family $\mathcal{F}$ of continuous contraction mappings on S where for each $U \in \tau$ we get $f(U) \subseteq U$ and $f(U) \in \tau'$.

Note the monster-barring of Lakatosian heuristics!

Results

Possible world semantics interpreted in topological spaces give us hints on using function for dynamic aspects of epistemology: learning, forgetting, updating, announcing, interacting etc.

We gave the semantics and referred to our own work following the Lakatosian path of heuristics with a dynamic modal touch.

Some Problems

Anachronism

Plato never meant neither of these!

Logicism

How can we know that all these math is true!

Non-working math

Proofs that do not prove are epistemologically valuable, but not mathematically.

Future Research

Logic of Heuristics

I am trying to develop a modal logic of (Lakatosian) heuristics. I welcome comments and criticism.

Computational Ethics

Meno is about the epistemic roots of ethics. Can we thus **compute** the virtue? Can there be non-computable virtues? What about the Turing degrees of virtues?

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Thanks!

Questions or Comments?

Talk slides and papers are available at:

www.canbaskent.net