Open Questions in Non-Classical Logic, Mathematics and Philosophy

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Outlook of the Talk

- Badia
- Başkent
- Behounek
- Bridges
- Garden
- Johnson
- Mares
- Priest
- Verdée

Goldblatt and Thomason provided model-theoretic characterizations of the classes of Kripke frames which are definable by a set of modal formulas and also the elementary classes of Kripke frames which are so definable using the constructions of bounded morphic images, subframes, disjoint unions and prime filter extensions. Can we obtain similar results in the context of relevant logic and Routley-Meyer frames for the relevant logic B? The presence of an actual world in these frames makes the usefulness of subframes, disjoint unions and prime filter extensions less transparent (for example, disjoint unions are not even well-defined in this context and prime filter extensions can only be constructed in some cases).

Strategic games can be defined using circular and inconsistent preferences. Would it generate *meaningful* equilibria in games?

When the agents have inconsistent preferences, inconsistent knowledge or inconsistent behavior, is it possible for the game to have *meaningful* equilibria? What are the game theoretical properties of such equilibria?

Is it possible to explore rational but inconsistent strategic behavior using paraconsistent logic or mathematics?

Behounek

Conjecture (Skolem, 1957): Naive comprehension is consistent over Łukasiewicz logic.

Despite some partial results, the problem is still open (Terui 2009 found a gap in White?s 1979 proof). More generally, it is unknown if naive comprehension is consistent over MTL = intuitionistic affine linear logic (aka FLew, where it is consistent) + prelinearity $(p \to q) \vee (q \to p)$, or any logics between MTL and Łukasiewicz logic.

Bridges

In his 1967 monograph, Errett Bishop succeeded in developing large parts of abstract analysis by fully constructive means (i.e. using intuitionistic logic and an informal constructive set theory). His work took in measure theory, the fundamentals of functional analysis, Haar measure on locally compact groups, and commutative Banach algebras. He showed clearly that constructive methods can be applied successfully even in the most abstract settings (something that has yet to be convincingly demonstrated for such classical-logic-based approaches as proof mining). Since then, several of us working in Bishop-style constructive mathematics (BISH) have produced more results in functional analysis, Banach algebras, and operator algebras. But the last two areas have barely been entered and remain open for detailed exploration. In particular, operator algebras, whose classical development relies heavily on nonconstructive arguments, remains as a major challenge for practitioners of BISH (and of all other approaches to (constructivity/computability in analysis).

Garden

The distributive lattice with orthogonal and complement is an interesting generalisation of a Boolean algebra and needs further study. This is the representation of Łukasiewicz **Ł3** and expresses all the Boolean laws of reasoning in the most general way. It has more interest than the term “quasi-Boolean” suggests and is at least *locally* Boolean in the sense that every element has an orthogonal-complement relative to a subsystem of the algebra, and so has a natural sense of logical relevance. What else?

Logic of (relative) infinitesimals is: (weakly) paraconsistent, purely iconic (thus requiring diagrammatic/ categorical techniques), primitively modal (i.e., irreducible to quantification, either first-order or second-order), "one-valued" (in the sense of my talk), and most likely involving nilpotent elements, i.e., non-zero objects such that $X \circ X = 0$ (or perhaps elements of higher periodicity than 2). Cathy Legg has recently been voicing the same Peircean pleas, reminding us all of another important feature, easily left out due to its superficially bizarre nature: for Peirce, any actualized point of the continuum can burst into a series of infinitesimals, all of which somehow “compose” the actual point. My hunch is that there already exists in the paraconsistent corpus, hidden somewhere, a rigorous way to make sense of this. For example, does Mortensen’s "ring of fire" model in the context of the paraconsistent closed set logic P3, wherein one finds boundaries made of non-self-identical objects, offer a possible way in?

Meyer showed that the relevant version of Peano arithmetic can be proven to be consistent using elementary (finitary) means. What does this entail about the provability/meaning of Goedel's second theorem for this system?

Meyer and Friedman showed that relevant peano arithmetic does not contain all of classical peano arithmetic on the natural translation. What axioms should be added to the system so that it will contain all of classical PA?

One of the main open problems of paraconsistent mathematics concerns paraconsistent set theory with an unrestricted comprehension schema. One strategy for approaching this topic (Weber's) is to take the theory to be based on an appropriate underlying relevant logic, the conditional of which is used in the comprehension schema. But another strategy is to take the theory to be formulated in an entirely extensional language, where the comprehension schema is formulated with a material biconditional. This theory is known to have a vast array of models, many with very interesting properties (such as validating all the theorems of ZF). Very little is known about the general structure of this space of models, however. In this way, the situation is quite unlike that concerning inconsistent arithmetic, where the structure of its space of models is generally very well understood. It is necessary to be much clearer about the structure of this space to pursue this approach to paraconsistent set theory.

Verdée

There has been little work on how exactly the formal systems of relevance logic relate to concepts in philosophy of science and philosophy of language (explanation, causation, diagnosis, belief revision, argumentation) for which relevance is an essential property of the involved deductive relations. It is, for example, quite uncontroversially clear that

* a cause has to be relevant for its effect,

* an explanans has to be relevant for its explanandum,

* an argument or a rebuttal has to be relevant for its conclusion,

* a counterfactual hypothesis has to be relevant for its consequent,

* a diagnosis has to be relevant for the diagnosed problem,

* in the context of a belief revision, contracted/removed information has to be relevant for the information with which beliefs are updated

However, the orthodox philosophical characterizations of these concepts provide only ad hoc solutions for such relevance problems or reduce them to mere "pragmatics". It is an important open problem how relevance logic could provide a more general and unifying account of relevance issues in philosophy.

Thank you!

Talk slides are available at:

`www.CanBaskent.net/Logic`