All Normal Extensions of **S5²** are Finitely Axiomatizable

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Road-Map and Recall Road-Map Recall

Mathematical Tools

p-morphism How to proceed?

Proof

Axiomatizability BQO-Theory comes in Results Related Results

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Complexity Results

Some Facts

-Road-Map and Recall

-Road-Map

Road-Map For the Proof

Recap

Necessary mathematical machinery

- Proof in several steps
- Complexity results

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- Proof in several steps
- Complexity results

Road-Map and Recall

Recall

Axioms of S5²

All tautologies of propositional calculus

- $\blacktriangleright \Box_i(\varphi \to \psi) \to (\Box_i \varphi \to \Box_i \psi)$
- $\blacktriangleright \Box_i \varphi \to \varphi$
- $\blacktriangleright \Box_i \varphi \to \Box_i \Box_i \varphi$
- $\blacktriangleright \Diamond_i \Box_i \varphi \to \varphi$
- $\blacktriangleright \Box_1 \Box_2 \varphi \leftrightarrow \Box_2 \Box_1 \varphi$

Two modal operators: \Box_1 and \Box_2 Closed under MP and Necessitation (from φ infer $\Box_i \varphi$).

Road-Map and Recall

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$$\blacktriangleright \Box_{i}(\varphi \to \psi) \to (\Box_{i}\varphi \to \Box_{i}\psi)$$

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Two modal operators: \Box_1 and \Box_2 Closed under MP and Necessitation (from φ infer $\Box_i \varphi$).

-Road-Map and Recall

- Recall

Facts on $S5^2(1)$

Complete with respect to $\{n \times n : n \ge 1\}$, for natural number *n* [Segerberg].

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where we have:

 $(x_1, x_2)R_1(y_1, y_2)$ iff $x_2 = y_2$

 $(x_1, x_2)R_2(y_1, y_2)$ iff $x_1 = y_1$

-Road-Map and Recall

Recall



Every proper extension *L* of **S5**² has poly-size model property; that is, there is a polynomial P(n) such that any *L*-consistent formula φ has a model over a frame validating *L* with at most $P(|\varphi|)$ points, where $|\varphi|$ is the length of the formula φ .

-Road-Map and Recall

Recall

Facts on $S5^2$ (3)

- $\mathcal{F} = (W, R_1, R_2)$ is a **S5²** frame where:
 - W is non-empty
 - \triangleright R_i 's are equivalence relations on W such that

 $\mathcal{F} \models (\forall w, v, u)(wR_1v \land vR_2u) \rightarrow (\exists z)(wR_2z \land zR_1u)$

Mathematical Tools

└*p*-morphism

p-morphism

For two **S5**² frames $\mathcal{F} = (W, R_1, R_2)$ and $\mathcal{G} = (U, S_1, S_2)$, *p*-morphism $f : U \to W$ from \mathcal{G} to \mathcal{F} , for each i = 1, 2 is defined as follows:

$$(\forall t \in U)(\forall w \in W)(f(t)R_iw \leftrightarrow (\exists u \in U)(tS_iu \land f(u) = w))$$

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-Mathematical Tools

p-morphism

Definitions on *p*-morphism

 $\mathbf{S5}^2$ frames \mathcal{F} is rooted if and only if

 $(\forall w, v)(\exists u)(wR_1u \wedge uR_2v))$

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Then define F_{S5^2} as the set of representatives of the isomorphism types of the finite rooted $S5^2$ frames. We will, from now on, consider the frames in F_{S5^2} . Why?

-Mathematical Tools

p-morphism

Definitions on *p*-morphism: an earlier result

Let *L* be a normal extension of **S5**². $\mathcal{F} \in \mathbf{S5}^2$ is called *L*-frame if \mathcal{F} validates each formula in *L*. Then, define \mathbf{F}_L the set of all *L*-frames in $\mathbf{F}_{\mathbf{S5}^2}$.

Bezhanisvili proved somewhere else that: L is complete wrt \mathbf{F}_{L} .

This is the reason why we will only consider the frames in F_{S5^2} . This is the first step towards our aim.

Define $\mathbf{M}_{\mathbf{L}} = min(\mathbf{F}_{\mathbf{S5}^2} \setminus \mathbf{F}_{\mathbf{L}}).$

Mathematical Tools

p-morphism

Definitions on *p*-morphism: a relation

We will introduce our first partial order in F_{S5^2} : \leq . For ${\cal F}$ and ${\cal G}$ in $F_{S5^2},$

 $\mathcal{F} \leq \mathcal{G}$ iff \mathcal{F} is a *p*-morphic image of \mathcal{G} .

For each \mathcal{G} in a subset A of $\mathbf{F}_{\mathbf{S5}^2}$, there is a frame $\mathcal{F} \in min(A)$ such that $\mathcal{F} \leq \mathcal{G}$.

-Mathematical Tools

-How to proceed?

Road-Map for the Proof

We will proceed as follows:

- Find a set of formulas that axiomatize any proper normal extension of S5².
- Show that this set is finite by stating equivalent statement about the finiteness of the set of axioms.

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Proof

Axiomatizability

Jankov-Fine Formulas

$$\alpha(\mathcal{F}) = \Box_{1} \Box_{2} \left(\bigvee_{\substack{p \in W}} (p \land \neg \bigvee_{p' \in W \setminus \{p\}} p' \land \bigwedge_{\substack{i=1,2\\p,p' \in W\\pR_{i}p'}} (p \to \Diamond_{i}p') \land \bigwedge_{\substack{i=1,2p\\p' \in W\\\neg(pR_{i}p')}} (p \to \neg \Diamond_{i}p') \right)$$

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- Axiomatizability

Why on earth do we need that formula?

• $\mathcal{F} \leq \mathcal{G}$ if and only if $\mathcal{G} \nvDash \chi(\mathcal{F})$.

- ▶ $\mathcal{G} \in \mathbf{F}_{L}$ if and only if for no $\mathcal{F} \in \mathbf{M}_{L}$, $\mathcal{F} \leq \mathcal{G}$, where $\mathbf{M}_{L} = min(\mathbf{F}_{\mathbf{S5}^{2}} \setminus \mathbf{F}_{L})$.
- ► Theorem Every proper normal extension *L* of S5² is axiomatizable by the axioms of S5² and {χ(𝒫) : 𝒫 ∈ M_L}.

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All Normal Extensions of S5<sup>2</sup> are Finitely Axiomatizable
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A qo-set (1)

R_i – **Depth** of \mathcal{F} is the number of R_i -equivalence classes of \mathcal{F} . Denote $d_i(\mathcal{F})$.

n(L) is the least *n* such that, $n \times n \notin \mathbf{F}_{L}$.

- ▶ If $\mathcal{F} \in \mathbf{F}_{\mathsf{L}}$, then $d_1(\mathcal{F}) < n(L)$ or $d_2(\mathcal{F}) < n(L)$.
- In contrast, if *F* is not in F_L, i.e. *F* ∈ M_L; then d₁(*F*) ≤ n(L) or d₂(*F*) ≤ n(L).
- ▶ The previous two results give rise to the following fact: $\mathbf{M}_{\mathbf{L}}$ is finite iff { $\mathcal{F} \in \mathbf{M}_{\mathbf{L}} : d_i(\mathcal{F}) = k$ } is finite for each $k \leq n(L)$ where i = 1, 2.

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All Normal Extensions of \mathbf{S5}^{\mathbf{2}} are Finitely Axiomatizable \hfill \mathsf{Proof}
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BQO-Theory comes in

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- ▶ The previous two results give rise to the following fact: $\mathbf{M}_{\mathbf{L}}$ is finite iff { $\mathcal{F} \in \mathbf{M}_{\mathbf{L}} : d_i(\mathcal{F}) = k$ } is finite for each $k \leq n(L)$ where i = 1, 2.



A qo-set (2)

So to prove the finiteness of \mathbf{M}_{L} , prove the finiteness of $\{\mathcal{F} \in \mathbf{M}_{\mathsf{L}} : d_i(\mathcal{F}) = k\}$ for each *k* while i = 1, 2.

But, since M_L is a \leq -antichain in F_{S5^2} , *instead* show M_L does *not* contain an infinite \leq -antichain.

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All Normal Extensions of S5<sup>2</sup> are Finitely Axiomatizable
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A qo-set (3): A Newer Relation

Fix *k*. WLOG, let i = 2. Let \mathcal{M}_n be the set of $n \times k$ matrices (m_{ij}) and \mathcal{M} is the collection $\bigcup_{n \in \omega} \mathcal{M}_n$.

 $(m_{ij}) \preceq (m'_{ij})$ holds if $(m_{ij}) \in \mathcal{M}_n$ and $(m'_{ij}) \in \mathcal{M}_{n'}$ and $n \leq n'$ and there is a surjection $f : n' \to n$ such that $m_{f(i)j} \leq m'_{ij}$.

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Observe that (\mathcal{M}, \preceq) is a qo-set.

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All Normal Extensions of \mathbf{S5}^2 are Finitely Axiomatizable
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BQO-Theory comes in

A qo-set (4)

Define $H : \mathbf{F}_{S5^2}^k \to \mathcal{M}$ by $H(\mathcal{F}) = (m_{ij})$, if $|F_i \cap F^j| = m_{ij}$. *H* is an order-reflecting injection, where $\mathbf{F}_{S5^2}^k$ is the set of frames in \mathbf{F}_{S5^2} with R_2 -depth *k*, F_i is the *i*th equivalence class of R_1 and F^j is the *j*th equivalence class of R_2 .

Therefore, for each \leq -antichain Δ in $\mathbf{F}_{\mathbf{S5}^2}^{\mathbf{k}}$, then $H(\Delta)$ is a \leq -antichain.

So, *instead*, we will show there is no infinite \leq -antichains in \mathcal{M} . But, *instead* of dealing with \leq , we will define new a quasi-order: \sqsubseteq .

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All Normal Extensions of S5<sup>2</sup> are Finitely Axiomatizable
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A qo-set (5): The Newest Relation

For $(m_{ij}) \in \mathcal{M}_n$ and $(m'_{ij}) \in \mathcal{M}_{n'}$:

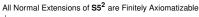
 $(m_{ij}) \sqsubseteq_1 (m'_{ij})$ if there is an injective order-preserving map $\varphi : n \to n'$ such that $m_{ij} \le m'_{\varphi(i)j}$ for each i < n and j < k.

 $(m_{ij}) \sqsubseteq_2 (m'_{ij})$ if there is a map $\psi : n' \to n$ such that $m_{\psi(i)j} \le m'_{ij}$ for each i < n and j < k.

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 \sqsubseteq is the intersection of \sqsubseteq_1 and \sqsubseteq_2 .

Thus, if $(m_{ij}) \sqsubseteq (m'_{ij})$, then $(m_{ij}) \preceq (m'_{ij})$.



Proof

BQO-Theory comes in



Therefore, *instead*, we will show there is no infinite \Box -antichains in \mathcal{M} .

FACT: There is no infinite antichains in a BQO.

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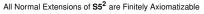
BQO-Theory comes in

BQOs: recap

► (ω, ≤) is a BQO.

- Any suborder of a BQO, and the intersection of two BQOs are BQOs.
- If (Q, \leq) is a BQO, then, $(\wp(Q), \leq)$ is a BQO.
- If (Q, ≤) is a BQO, then (⋃_{α∈On} Q^α, ≤*) is a BQO. Hence, the suborders (Q^k, ≤*) and ⋃_{n<ω} Qⁿ, ≤* are BQOs.

Define \leq^* on the class $\bigcup_{\alpha \in On} Q^{\alpha}$ by $(x_i)_{i < \alpha} \leq^* (y_i)_{i < \beta}$ if there is an order-preserving map $\varphi : \alpha \to \beta$ such that $x_i \leq y_{\varphi(i)}$ for each $i < \alpha$.

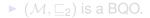


Proof

Results

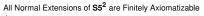


▶ $(\mathcal{M}, \sqsubseteq_1)$ is a BQO.









L Proof

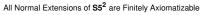
Results



•
$$(\mathcal{M}, \sqsubseteq_1)$$
 is a BQO.

▶ Thus, $(\mathcal{M}, \sqsubseteq)$ is a BQO.

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L_Proof

Results



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$$(\mathcal{M}, \sqsubseteq_1)$$
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All Normal Extensions of $\mathbf{S5}^2$ are Finitely Axiomatizable

Proof

Results

Result (theorem)

THEOREM: All normal extensions of **S5**² are finitely axiomatizable.

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- Results

Result (proof)

S5² is finitely axiomatizable.

- If *L* is a proper extension of S5², then it is axiomatizable by the axioms of S5² and {χ(F) : F ∈ M_L}.
- Since ⊑ is a BQO, it has no ⊑-infinite antichains, and there is no ∠-antichains in M.
- Therefore for each k ∈ ω, F^k_{S5²} has no infinite antichains. Thus, for each k ≤ n(L), the set {F ∈ M_L : d_i(F) = k} has finite number of elements.
- ► Hence, $\mathbf{M}_{\mathbf{L}}$ is finite, and there are only finitely many $\chi(\mathcal{F})$ formulas that axiomatize *L*.

- Results

- ▶ **S5**² is finitely axiomatizable.
- If L is a proper extension of S5², then it is axiomatizable by the axioms of S5² and {χ(F) : F ∈ M_L}.
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- Therefore for each k ∈ ω, F^k_{S5²} has no infinite antichains. Thus, for each k ≤ n(L), the set {F ∈ M_L : d_i(F) = k} has finite number of elements.
- ► Hence, M_L is finite, and there are only finitely many χ(F) formulas that axiomatize L.

-Complexity Results

-Some Facts

SAT

- S5² has a exponential size model property, and its satisfiability problem is NEXP-TIME.
- Every proper normal extension of S5² is decidable in polynomial time. Therefore, together with the poly-size model property, it implies that the satisfiability for the normal proper extension is NP-complete.

POLY-SIZE MODEL PROPERTY For the each proper normal extension *L* of **S5**², there is a polynomial P(n) s.t. for any *L*-consistent formula ϕ has a model over a frame validating *L*, and model has at most $P(|\phi|)$ points where $P(|\phi|)$ denotes the length of ϕ .

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Thanks for your attention

