Syllogisms in Aristotle and Boethius

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Categorical Syllogism in Aristotle
- Definitions
- Figures of Categorical Syllogism

Hypothetical Syllogism in Aristotle
- Hints in Texts

Categorical Syllogism in Boethius
- Boethius' Definitions
- Alterations

Hypothetical Syllogism in Boethius

Further Works

Syllogisms in Aristotle

Back to the Basics

- All philosophers are mortal. [Major Premise]
- Socrates is a philosopher. [Minor Premise]
- Therefore, Socrates is mortal. [Conclusion]

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Wiki definition

Categorical syllogism, is a kind of logical argument in which one proposition (the conclusion) is inferred from two others (the premises).

Aristotle's Definition and Critics

" a discourse in which, certain things being stated, something other than what is stated follows of necessity from their being so.. I mean by the last phrase that they produce the consequence, and by this, that no further term is required from without to make the consequence necessary."
from *Prior Analytics*

Aristotle's Definition and Critics

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Rusinoff: This definition does not distinguish syllogism from other forms of inference.

Categorical Senteces

(A) A belongs to all B. (AaB)
(I) A belongs to some B. (AiB)
(E) A does not belong to any B. (AeB)
(O) A does not belong to some B. (AoB)

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Three Figures

$$
\begin{array}{ccc}
\text{I.} & \text{II.} & \text{III.} \\
A - B & B - A & A - B \\
\underline{B - C} & \underline{B - C} & \underline{C - B} \\
A - C & A - C & A - C
\end{array}
$$

where A is the major, B is the middle and C is the minor term.

Conversion Rules

- $A\mathrm{a}B \Rightarrow B\mathrm{i}A$.
- $A\mathrm{i}B \equiv B\mathrm{i}A$.
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Four Figures - 1

First Figure

Barbara	Celarent	Darii	Ferio
AaB	AeB	AaB	AeB
$\underline{BaC}$	$\underline{BaC}$	$\underline{BiC}$	$\underline{BiC}$
AaC	AeC	AiC	AoC

Second Figure

Camestres	Cesare	Festino	Baroco
BaA	BeA	BeA	BaA
$\underline{BeC}$	$\underline{BaC}$	$\underline{BiC}$	$\underline{BoC}$
AeC	AeC	AoC	AoC

Four Figures - 2

Third Figure

Darapti	Felapton	Disamis	Datisi	Bocardo	Ferison
AaB	AeB	AiB	AaB	AoB	AeB
$\underline{CaB}$	$\underline{CaB}$	$\underline{CaB}$	$\underline{CiB}$	$\underline{CaB}$	$\underline{CiB}$
AiC	AoC	AiC	AiC	AoC	AoC

Fourth Figure [not mentioned in Aristotle explicitly]

Bramantip	Camenas	Dimaris	Fesapo	Fresison
BaA	BaA	BiA	BeA	BeA
$\underline{CaB}$	$\underline{CeB}$	$\underline{CaB}$	$\underline{CaB}$	$\underline{CiB}$
AiC	AeC	AiC	AoC	AoC

First Figure

Barbara	Celarent	Darii	Ferio
$\dfrac{AaB \quad BaC}{AaC}$	$\dfrac{AeB \quad BaC}{AeC}$	$\dfrac{AaB \quad BiC}{AiC}$	$\dfrac{AeB \quad BiC}{AoC}$

First Figure as a Set of Axioms

Barbara	Celarent	Darii	Ferio
$\dfrac{A\,a\,B \quad B\,a\,C}{A\,a\,C}$	$\dfrac{A\,e\,B \quad B\,a\,C}{A\,e\,C}$	$\dfrac{A\,a\,B \quad B\,i\,C}{A\,i\,C}$	$\dfrac{A\,e\,B \quad B\,i\,C}{A\,o\,C}$

- First figure was evidently clear for Aristotle. *Nothing needs to be added to make it more evident.*
- No proof for the first figure was given.
- Reduced other figures to the first figure.

First Figure as a Set of Axioms

Barbara	Celarent	Darii	Ferio
$\dfrac{\begin{array}{c} AaB \\ BaC \end{array}}{AaC}$	$\dfrac{\begin{array}{c} AeB \\ BaC \end{array}}{AeC}$	$\dfrac{\begin{array}{c} AaB \\ BiC \end{array}}{AiC}$	$\dfrac{\begin{array}{c} AeB \\ BiC \end{array}}{AoC}$

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Excerpt from *Prior Analytics*

- It is possible that the premises from which the syllogism is formed are true; and it is possible, likewise, that they are false, or that one is true and the other false. The conclusion is necessarily either true or false.
- If two things are related to each other in such a way that the existence of one entails necessarily the existence of the other, [then] the non-existence of the last one will entail the non-existence of the first.
- It is impossible that B should necessarily be great since A is white and that B should necessarily be great since A is not white. For whenever since this, A, is white it is necessary that, B, should be great, and since B is great that C should not be white, then it is necessary if is white that C should not be white.

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Interpretation

- From true premises, one cannot draw a false conclusion, but from false premises one can draw a true conclusion
- If when A is, B must be, then when B is not, necessarily A cannot be.
- If from A follows necessarily B, and from B follows non-C, then necessarily from A follows non-C.

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Opinions on the relation between HS and Aristotle - 1

Dumitriu (*History of Logic*): Aristotle did *not* develop a theory of HS. For Aristotle, reasoning must lead to necessary conclusions, not to *per accidens* conclusions.

Kneale and Kneale (*The Development of Logic*): Aristotle did not recognize the conditional form of statement and argument based on it as an object of logical inquiry.

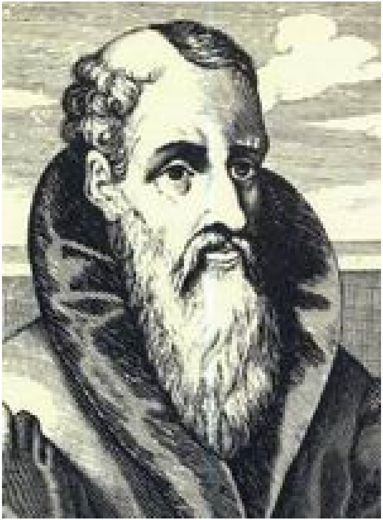
Opinions on the relation between HS and Aristotle - 2

Stoics, having nominalist concept of truth, studied HS extensively.

Peripatetic School also studied HS extensively.

Theophrastus and Eudemus were the leading figures.

Syllogisms in Anicius Manlius Severinus Boethius



Belong versus Is

(A') Every B is A.

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(E') No B is A.

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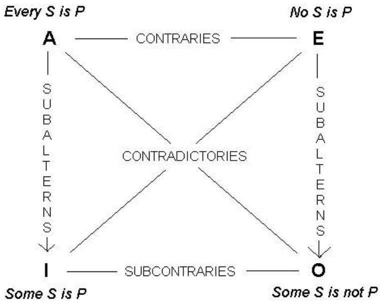
Square of Opposition



Aristotle vs Boethius: Categorical Sentences

Aristotle	**Boethius**
A belongs to all B	Every B is A.
A belongs to some B	Some B is A.
A does not belong to any B	No B is A.
A does not belong to some B	Some B is not A.

Critics - 1

Boethius was accused of obscuring the theory of the syllogism, since his translation of belong to is, is claimed to make it unclear why the first figure (of Aristotle) was evident and was not in need of a proof.

Critics - 2

Boethius added a fourth conversion rule: as universal affirmative can be converted to particular affirmative, universal negative can be converted to particular negative:

$$A\mathrm{e}B \Rightarrow A\mathrm{o}B$$

Critics - 3

Boethius' Four Categorical Sentences

$$\begin{array}{cccc}
\text{I.} & \text{II.} & \text{III.} & \text{IV.} \\
B \text{ - } A & A \text{ - } B & B \text{ - } A & A \text{ - } B \\
\underline{C \text{ - } B} & \underline{C \text{ - } B} & \underline{B \text{ - } C} & \underline{B \text{ - } C} \\
C \text{ - } A & C \text{ - } A & C \text{ - } A & C \text{ - } A
\end{array}$$

Categorical Sentences: Aristotle vs. Boethius

Boethius' Four Categorical Sentences

$$\begin{array}{cccc} \text{I.} & \text{II.} & \text{III.} & \text{IV.} \\ B - A & A - B & B - A & A - B \\ \underline{C - B} & \underline{C - B} & \underline{B - C} & \underline{B - C} \\ C - A & C - A & C - A & C - A \end{array}$$

Aristotle's Three Categorical Sentences

$$\begin{array}{ccc} \text{I.} & \text{II.} & \text{III.} \\ A - B & B - A & A - B \\ \underline{B - C} & \underline{B - C} & \underline{C - B} \\ A - C & A - C & A - C \end{array}$$

Introduction - 1

- Boethius extended and enlarged Aristotle's works on HS.
- "Devoted a lot of his time to a tiresome but efficient work" on this.
- For this reason he was considered for a long time as the discoverer of HS.

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- Draws distinction between categorical sentences and hypothetical sentences.
- Relates the theory of HS with Theopharastus and Eudeomos.
- Claimed "Aristotle wrote nothing" on HS, could not find any representation of HS in Latin scholars.

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Hypothetical Syllogism for Boethius - 1

Two kinds of hypothetical sentence: *simple* and *complex*. Simple ones are of the form "If A is, then B is" whereas the complex ones are of the form "If A is, then, in case B is, C is too". He gave the four possible examples:

1. "If it is day, it is light".
2. "If it is not an animal, it is not a man."
3. "If it is day, it is not night."
4. "If it is not day, it is night."

Hypothetical Syllogism for Boethius - 2

Distinguished between *perfect* and *imperfect* HS.

Perfect HS requires no demonstration whereas imperfect one needs a demonstration.

Hypothetical Syllogism for Boethius - 3

Boethius considered "accidental" conditionals and gave the following example:

> *If fire is hot, the heavens are spherical.*

It is clear that the statement is true, as both the antecedant and concequent are true. However, there is no relation between what both sentence talk about. This is what makes this kind of sentences *accidental*.

Algebraizing Syllogisms

Susan Russinof in discussed syllogisms in an algebraic setting and gives an algebraic interpretation of categorical statements quoting Christine Ladd-Franklin's 1883 paper:

(A) $A\mathrm{a}B$ becomes $A < B$ or $A - B = 0$

(I) $A\mathrm{i}B$ becomes $A < -B$ or $A.B = 0$

(E) $A\mathrm{e}B$ becomes $(A < -B)'$ or $A.B \neq 0$

(O) $A\mathrm{o}B$ becomes $(A < B)'$ or $A - B \neq 0$

Reference: RUSSINOFF, I . S.: *The syllogisms final solution.* The Bulletin of Symbolic Logic vol. 5, no. 4 (December 1999), pp. 45169.

Thanks for your attention