A Yabloesque Paradox in Epistemic Game Theory

where paraconsistency meets game theory

Can Başkent

Department of Computer Science, University of Bath
can@canbaskent.net www.canbaskent.net/logic @topologically

Logic and the Foundations of Game and Decision Theory (LOFT 2015) - July 20-22, 2016

Today's Plan

Slogan: Paraconsistency for Game Theory!

Paraconsistent Game Theory

helps us understand games

and paradoxes better!

Paradoxes

Yablo's Paradox

Yablo's Paradox, according to its author, is a non-self referential paradox (Yablo, 1985; Yablo, 1993).

Yablo considers the following sequence of sentences.

$$
\begin{aligned}
S_1 &: \forall k > 1, S_k \text{ is untrue,} \\
S_2 &: \forall k > 2, S_k \text{ is untrue,} \\
S_3 &: \forall k > 3, S_k \text{ is untrue,} \\
&\vdots
\end{aligned}
$$

By using *reductio*, Yablo argues that the above set of sentences is contradictory. Here, the infinitary nature of the paradox is essential as the each finite set of S_n is satisfiable.

The scheme of this paradox is not new. To the best of our knowledge, the first analysis of this paradox was suggested in 1953 (Yuting, 1953).

Impact of Yablo's Paradox

Ketland showed that the paradox is ω-inconsistent (Ketland, 2005).

Barrio showed that Yablo's Paradox in first-order arithmetic has a model and not inconsistent, but it is ω-inconsistent (Barrio, 2010).

Goldstein presents a set theoretical yabloesque paradox for class membership (Goldstein, 1994).

Picollo discusses the paradox in second-order logic generalizing the ω-inconsistency results (Picollo, 2013).

Non-well-founded Yablo chains form a topological space encouraging Bernardi's topological approach to the paradox (Bernardi, 2009).

Cook and Beall consider Curry-like versions of the paradox (Cook, 2009; Beall, 1999).

Bridge towards Epistemic Games

Can we apply Yablo's arguments to epistemic game theory?

Let us start with the Brandenburger - Keisler paradox.

Brandenburger - Keisler Paradox

The Brandenburger-Keisler paradox is a two-person self-referential paradox in epistemic game theory (Brandenburger & Keisler, 2006).

The following configuration of beliefs is impossible:

The Paradox

Ann believes that Bob assumes that Ann believes that Bob's assumption is wrong.

The paradox appears if you ask whether "Ann believes that Bob's assumption is wrong".

Notice that this is essentially a 2-person Russell's Paradox.

Brandenburger - Keisler Paradox

Brandenburger and Keisler showed that no belief model is complete for its (classical) first-order language.

Therefore, "not every description of belief can be represented" with belief structures (Brandenburger & Keisler, 2006).

Yabloesque Paradox in Epistemic Games

Consider the following sequence of assumptions where numerals represent game theoretical players.

$$A_1 : 1 \text{ believes that } \forall k > 1, k\text{'s assumption } A_l \text{ about } \forall l > k \text{ is untrue,}$$
$$A_2 : 2 \text{ believes that } \forall k > 2, k\text{'s assumption } A_l \text{ about } \forall l > k \text{ is untrue,}$$
$$A_3 : 3 \text{ believes that } \forall k > 3, k\text{'s assumption } A_l \text{ about } \forall l > k \text{ is untrue,}$$
$$\vdots$$

An Interpretation

Imagine a queue of players, where players are conveniently named after numerals, holding beliefs about each player behind them, but not about themselves. In this case, each player i believes that each player $k > i$ behind them has an assumption about each other player $l > k$ behind them and i believes that each k's assumption is false.

This statement is perfectly perceivable for games, and involves a specific configuration of players' beliefs and assumptions, which can be expressible in the language. However, as we shall show, similar to Yablo's paradox and the Brandenburger - Keisler paradox, this configuration of beliefs is impossible.

The Yabloesque Brandenburger - Keisler paradox requires ω-many players $i \in I$. The language is given as follows for a set of propositional variables P:

$$\varphi := p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \Box^{ij}\varphi \mid \heartsuit^{ij}\varphi$$

where $p \in \mathsf{P}$ and $i \neq j$ for $i, j \in I$ with $|I| = \omega$.

The extended belief model is a tuple $M = (\{U^i\}_{i \in I}, \{R^{ij}\}_{i \neq j \in I}, V)$ where $R^{ij} \subseteq U^i \times U^j$ and V is a valuation function.

The expression $R^{ij}(x, y)$ represents that in state x, the player i believes that the state y is possible for player j.

The semantics for the modal operators is given as follows.

$$x \models \Box^{ij}\varphi \quad \textit{iff} \quad \forall y \in U^j . R^{ij}(x,y) \textit{ implies } y \models \varphi$$
$$x \models \heartsuit^{ij}\varphi \quad \textit{iff} \quad \forall y \in U^j . R^{ij}(x,y) \textit{ iff } y \models \varphi$$

Theorem

The Yabloesque Brandenburger - Keisler paradox is paradoxical.

A Paraconsistent Model

For the same syntax, it is possible to give a topological semantics for epistemic game models that is *inconsistency-friendly*.

Therefore, there exists a *paraconsistent* model in which the yabloesque Brandenburger - Keisler paradox is satisfiable!

Conclusion

Conclusion

A self-referential paradox in games was already given.

In this paper, we give a *non*-self-referential paradox and show its inconsistency.

We also give a paraconsistent model for it (in the poster).

Thank you!

Come see the poster

Talk slides and the papers are available at

`CanBaskent.net/Logic`

References

Barrio, Eduardo Alejandro. 2010.
Theories of Truth without Standard Models and Yablo's Sequences.
Studia Logica, **96**(3), 375–391.

Beall, Jc. 1999.
Completing Sorensen's Menu: A Non-Modal Yabloesque Curry.
Mind, **108**(431), 737–739.

Bernardi, Claudio. 2009.
A Topological Approach to Yablo's Paradox.
Notre Dame Journal of Formal Logic, **50**(3), 331–338.

A Brief Bibliography II

Brandenburger, Adam, & Keisler, H. Jerome. 2006.
An Impossibility Theorem on Beliefs in Games.
Studia Logica, **84**, 211–240.

Cook, Roy T. 2009.
Curry, Yablo and Duality.
Analysis, **69**(4), 612–620.

Goldstein, Laurence. 1994.
A Yabloesque Paradox in Set Theory.
Analysis, **54**(4), 223–7.

Ketland, Jeffrey. 2005.
Yablo's Paradox and ω-Inconsistency.
Synthese, **145**(3), 295–302.

A Brief Bibliography III

Picollo, Lavinia María. 2013.
Yablo's Paradox in Second-Order Languages: Consistency and Unsatisfiability.
Studia Logica, **101**(3), 601–617.

Yablo, Stephen. 1985.
Truth and Reflection.
Journal of Philosophical Logic, **14**(3), 297–349.

Yablo, Stephen. 1993.
Paradox without Self-Reference.
Analysis, **53**(4), 251–2.

Yuting, Shen. 1953.
Paradox of the Class of All Grounded Classes.
The Journal of Symbolic Logic, **18**(2), 114.