

Paradoxes and Games



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where paraconsistency meets game theory

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Today's Plan

1. Quick Examples
2. Paradoxes
3. A Yabloesque Paradox in Epistemic Games

Slogan: Paraconsistency for Game Theory!

Paraconsistent Game Theory

helps us understand games

and paradoxes better!

Quick Examples

“Wardrobing”

“Wardrobing is buying an item of clothing, wearing it for a while, and then returning it in such a state that the store has to accept it but can no longer resell it. By engaging in wardrobing, consumers are not directly stealing money from the company; instead, it is a dance of buying and returning, with many unclear transactions involved. But there is at least one clear consequence — the clothing industry estimates that its annual losses from wardrobing are about \$16 billion (about the same amount as the estimated annual loss from home burglaries and automobile theft combined) (...) [This is] the level of dishonesty practiced by individuals who want to be ethical and who want to see themselves as ethical — the so-called good people.”

Predictably Irrational, Dan Ariely, Harper, 2008.

Arrow's Theorem

Paradoxes and impossibility results have shaped Social Choice Theory.

“The theorem states that no rank-order voting system can be designed that always satisfies these three ‘fairness’ criteria:

- If every voter prefers alternative X over alternative Y , then the group prefers X over Y .
- If every voter's preference between X and Y remains unchanged, then the group's preference between X and Y will also remain unchanged (even if voters' preferences between other pairs like X and Z , Y and Z , or Z and W change).
- There is no “dictator”: no single voter possesses the power to always determine the group's preference.

[from *Wikipedia*]

The Liberal Paradox

“The liberal paradox, also Sen paradox or Sen’s paradox, is a logical paradox discovered by Amartya Sen which purports to show that no social system can simultaneously

- be committed to a minimal sense of freedom,
- always result in a type of economic efficiency known as Pareto efficiency, and
- be capable of functioning in any society whatsoever.”

[from *Wikipedia*]

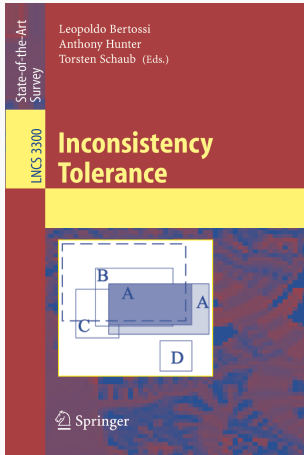
The Gibbard–Satterthwaite Theorem

“The Gibbard–Satterthwaite theorem states that, for three or more candidates, one of the following three things must hold for every voting rule:

- The rule is dictatorial (i.e., there is a single individual who can choose the winner), or
- There is some candidate who can never win, under the rule, or
- The rule is susceptible to tactical voting, in the sense that there are conditions under which a voter with full knowledge of how the other voters are to vote and of the rule being used would have an incentive to vote in a manner that does not reflect his or her preferences.

[from *Wikipedia*]

Inconsistency Tolerance



How can you handle inconsistencies, besides belief revision or dynamic updates?

Inconsistency Tolerance, L. Bertossi, A. Hunter, T. Schaub (editors), Springer, 2005.

Semantic Games

Hintikka semantic games are determined, two-player, sequential, zero-sum, non-cooperative as they are *designed* for classical logic.

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Semantic Games

Hintikkan semantic games are determined, two-player, sequential, zero-sum, non-cooperative as they are *designed* for classical logic.

- Can we design *logics for semantic games* with different properties? What is the logic of three-player, non-zero sum undetermined games?
- Can we design *games for non-classical logics*? How can we play semantic games for Belnap's B4 or da Costa Systems?

“Game Theoretical Semantics for Some Non-Classical Logic”, CB, *Journal of Applied Non-Classical Logics*, vol. 26, no. 3, pp. 208-39, 2016.

Paradoxes

Brandenburger - Keisler Paradox

The Brandenburger-Keisler paradox is a two-person self-referential paradox in epistemic game theory.

The following configuration of beliefs is impossible:

The Paradox

Ann believes that Bob assumes that Ann believes that Bob's assumption is wrong.

The paradox appears if you ask whether "Ann believes that Bob's assumption is wrong".

Notice that this is essentially a 2-person Russell's Paradox.

Brandenburger - Keisler Paradox

Brandenburger and Keisler showed that no belief model is complete for its (classical) first-order language: there are statements that cannot be modelled.

Therefore, "not every description of belief can be represented" with belief structures.

"An Impossibility Theorem on Beliefs in Games", A. Brandenburger and J. Keisler, *Studia Logica*, vol. 84, pp. 211-240, 2006.

Brandenburger - Keisler Paradox

This is a self-referential paradox. A two- (or n -) person generalization of the Liar Paradox, rendering it in an interactive and game theoretical setting.

Theoretical richness of self-referentiality relates it to fixed-points and a broad class of logics and models.

“Some Non-Classical Approaches to the Brandenburger-Keisler paradox”, CB, *Logic Journal of the IGPL*, vol. 23, pp. 533-552, 2015.

“From Lawvere to Brandenburger-Keisler: Interactive forms of diagonalization and self-reference”, S. Abramsky and J. Zvesper, *Journal of Computer and System Sciences*, vol. 81, pp. 799-812, 2015.

The Model

The *assumption* modality is worth exploring:

The language is given as follows for a set of propositional variables \mathbf{P} :

$$\varphi := p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \square^{ij}\varphi \mid \heartsuit^{ij}\varphi$$

where $p \in \mathbf{P}$ and $i \neq j$ for $i, j \in 2$. \square^{ij} denotes the knowledge modality whereas \heartsuit^{ij} is the assumption modality.

The belief model is a tuple $M = (U^i, R^{ij}, V)$ where $R^{ij} \subseteq U^i \times U^j$ and V is a valuation function.

The expression $R^{ij}(x, y)$ represents that in state x , the player i believes that the state y is possible for player j .

The semantics for the modal operators is given as follows.

$$\begin{array}{l} x \models \Box^{ij}\varphi \quad \text{iff} \quad \forall y \in U^j. R^{ij}(x, y) \quad \text{implies} \quad y \models \varphi \\ x \models \heartsuit^{ij}\varphi \quad \text{iff} \quad \forall y \in U^j. R^{ij}(x, y) \quad \text{iff} \quad y \models \varphi \end{array}$$

Assumption is strong belief, perhaps the strongest belief.

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Is there a *non*-self-referential *game theoretical* paradox?

Yablo's Paradox

Yablo's Paradox, according to its author, is a non-self referential paradox.

Yablo considers the following sequence of sentences.

$S_1 : \forall k > 1, S_k$ is untrue,

$S_2 : \forall k > 2, S_k$ is untrue,

$S_3 : \forall k > 3, S_k$ is untrue,

\vdots

"Paradox without Self-Reference", S. Yablo, *Analysis*, vol. 53, pp. 251-2, 1993.

Yablo's Paradox

By using *reductio*, Yablo argues that the above set of sentences is contradictory. Here, the infinitary nature of the paradox is essential as the each finite set of S_n is satisfiable.

The scheme of this paradox is not new. To the best of our knowledge, the first analysis of this paradox was suggested in 1953 by Yuting.

“Paradox of the Class of All Grounded Classes”, Sh. Yuting, *The Journal of Symbolic Logic*, vol. 18, p. 114, 1953.

A Yabloesque Paradox in Epistemic Games

Is there a *non-self-referential game theoretical* paradox?

A Yabloesque Paradox in Epistemic Games

Consider the following sequence of assumptions where numerals represent game theoretical players.

A_1 : 1 believes that $\forall k > 1$, k 's assumption A_l about $\forall l > k$ is untrue,

A_2 : 2 believes that $\forall k > 2$, k 's assumption A_l about $\forall l > k$ is untrue,

A_3 : 3 believes that $\forall k > 3$, k 's assumption A_l about $\forall l > k$ is untrue,

\vdots

“A Yabloesque Paradox in Epistemic Game Theory”, CB, *Synthese*, 2016, forthcoming (online first access).

An Interpretation

Imagine a queue of players, where players are conveniently named after numerals, holding beliefs about each player behind them, but not about themselves. In this case, each player i believes that each player $k > i$ behind them has an assumption about each other player $l > k$ behind them and i believes that each k 's assumption is false.

This statement is perfectly perceivable for games, and involves a specific configuration of players' beliefs and assumptions, which can be expressible in the language.

However, similar to Yablo's paradox and the Brandenburger - Keisler paradox, this configuration of beliefs is impossible.

The Model

The Yabloesque Brandenburger - Keisler paradox requires ω -many players $i \in I$. The language is given as follows for a set of propositional variables \mathbf{P} :

$$\varphi := p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \square^{ij}\varphi \mid \heartsuit^{ij}\varphi$$

where $p \in \mathbf{P}$ and $i \neq j$ for $i, j \in I$ with $|I| = \omega$.

The extended belief model is a tuple $M = (\{U^i\}_{i \in I}, \{R^{ij}\}_{i \neq j \in I}, V)$ where $R^{ij} \subseteq U^i \times U^j$ and V is a valuation function.

The expression $R^{ij}(x, y)$ represents that in state x , the player i believes that the state y is possible for player j .

The semantics for the modal operators is given as follows.

$$\begin{aligned}x \models \Box^{ij}\varphi & \text{ iff } \forall y \in U^j. R^{ij}(x, y) \text{ implies } y \models \varphi \\x \models \heartsuit^{ij}\varphi & \text{ iff } \forall y \in U^j. R^{ij}(x, y) \text{ iff } y \models \varphi\end{aligned}$$

Theorem

The Yabloesque Brandenburger - Keisler sentence is inconsistent.

A Paraconsistent Model

For the same syntax, it is possible to give a topological semantics for epistemic game models that is *inconsistency-friendly*.

Therefore, there exists a *paraconsistent* model in which the yabloesque Brandenburger - Keisler paradox is satisfiable!

A Paraconsistent Model

Paraconsistent models are not difficult to construct.

Topological models are the first ones for modal logic, suggested by McKinsey in 1941!

Topological semantics associates the extension of modal formulas with open (or closed) sets. It necessitates that the extension of modal formulas are topological sets. For ground formulas, there is no restriction.

A Paraconsistent Model

In intuitionistic logic, it is imposed that the extension of propositional variables are open sets.

Dually, in paraconsistent logic, it is imposed that they are closed sets.

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Dually, in paraconsistent logic, it is imposed that they are closed sets.

What about negation then?

A Paraconsistent Model

Since negation of a closed/open set is not closed/open, in non-classical logics, a new negation is introduced: *closed/open negation*.

In intuitionistic case, it is the *interior* of the complement,

In paraconsistent case, it is the *closure* of the complement.

A Paraconsistent Model

Since negation of a closed/open set is not closed/open, in non-classical logics, a new negation is introduced: *closed/open negation*.

In intuitionistic case, it is the *interior* of the complement,

In paraconsistent case, it is the *closure* of the complement.

In intuitionistic case, it generates truth value *gaps*,

In paraconsistent case, it generates truth value *gluts*.

“Some Topological Properties of Paraconsistent Models”, CB, *Synthese*, vol. 190, no. 18, pp. 4023-4040, 2013.

A Paraconsistent Model

Topological paraconsistent models work in epistemic game theory: they are portable, versatile and enjoy a broad theoretical background.

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Topological paraconsistent models work in epistemic game theory: they are portable, versatile and enjoy a broad theoretical background.

Using topological products, it is possible to construct interactive, multi-agent models for epistemic game theory that can allocate non-classical logics.

“Some Non-Classical Approaches to the Brandenburger-Keisler paradox”, CB, *Logic Journal of the IGPL*, vol. 23, pp. 533-552, 2015.

“A Yabloesque Paradox in Epistemic Game Theory”, CB, *Synthese*, 2016, forthcoming (online first access).

Paraconsistency for Game Theory!

and

Game Theory for Paraconsistency!

Paraconsistent Game Theory

helps us understand games

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Thank you!

Talk slides and the papers are available at

CanBaskent.net/Logic