A History Based Logic for Dynamic Preference Updates

Can Başkent

Department of Computer Science, Middlesex University
c.baskent@mdx.ac.uk canbaskent.net/logic @topologically

Today's Plan

- Joint work with Guy McCusker, University of Bath.

Slogan!

Rational agents

update

their preferences and strategies!

Introduction

Motivation

Games are processes. What happened in the past matters for the behaviour in the future.

Games have strategies which represent subjective preferences: often defined as fixed before the game.

What if players' preferences change? Players may receive new information, obligation or choose to revise their beliefs and preferences.

Motivation

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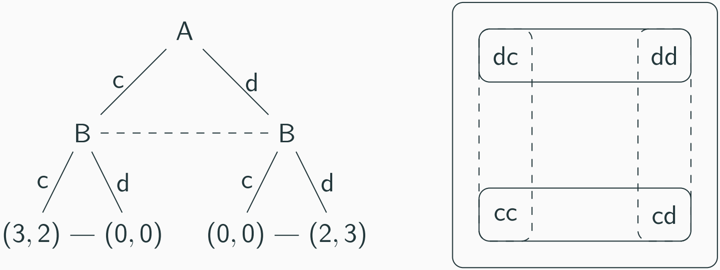
Challenge

How can we represent game theoretical preference changes using processes?

An Example

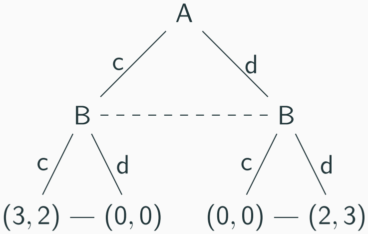


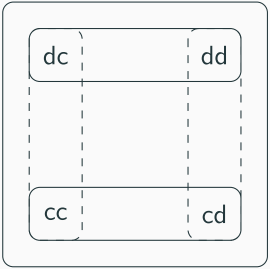
Battle of the Sexes in extensive normal form (left) and its epistemic indistinguishability relation.

The solid line defines the knowledge set of Player A whereas the dashed line defines that of B.

The utility pair (x, y) means that x is A's pay-off whereas y is that of B.

An Example





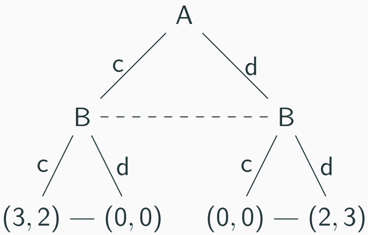
Player A's preferences can be represented as follows:

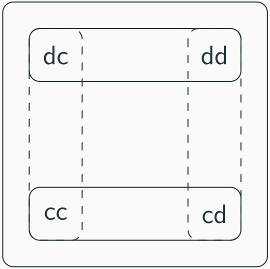
$$\mathsf{cd} \preceq_A \mathsf{dc} \preceq_A \mathsf{dd} \preceq_A \mathsf{cc}$$

and

$$\mathsf{cd} \sim_A \mathsf{cc} \qquad\qquad \mathsf{dd} \sim_A \mathsf{dc}.$$

An Example





What if a friend tells A that B is making a d move?

Player A **updates** his preferences to have

$$\text{cd} \preceq_A \text{dd}$$

The Formal Structure

Histories

Given a set of events E and a set of agents A:

For each agent $i \in \mathsf{A}$, $\mathsf{E}_i \subseteq \mathsf{E}$ is the set of events which can be performed or "seen" by agent i.

By $H, H', \ldots$, we denote histories from $\mathsf{E}^* \cup \mathsf{E}^\omega$, finite or infinite strings over E.

For a set of histories $\mathbf{H}$, let $FP(\mathbf{H})$ be the set of finite prefixes of the histories in $\mathbf{H}$. Then, $\lambda_i : FP(\mathbf{H}) \mapsto \mathsf{E}_i^*$ is a locality function to define epistemic indistinguishability of agent i with respect to $\mathbf{H}$.

For $H, H' \in \mathbf{H}$, if $\lambda_i(H) = \lambda_i(H')$, then we say H and H' are epistemically indistinguishable for agent i and denote it by $H \sim_i H'$.

Syntax

Given a set of propositional variables P, we define the syntax of history based structures as follows, for $p \in P$, $i \in \mathsf{A}$.

$$\varphi := \; p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid K_i\varphi \mid \bigcirc\varphi \mid \varphi U \varphi \mid \Diamond_i \varphi$$

The knowledge operator for agent i is denoted by K_i and the temporal next-time operator is denoted by $\bigcirc$. We call U the until operator and $\Diamond$ the preference operator.

History based preference model is a tuple

$$M = (\mathsf{E}, \mathcal{H}, \mathsf{A}, \mathsf{E}_1, \ldots, \mathsf{E}_n, \lambda_1, \ldots, \lambda_n, \preceq_1, \ldots, \preceq_n, V)$$

where V is a valuation in the usual sense.

Semantics

$$
\begin{array}{lll}
H, t \models p & \text{iff} & H_t \in V(p), \\
H, t \models \neg\varphi & \text{iff} & H, t \not\models_M \varphi, \\
H, t \models \varphi \wedge \psi & \text{iff} & H, t \models_M \varphi \text{ and } H, t \models_M \psi, \\
H, t \models \varphi \vee \psi & \text{iff} & H, t \models_M \varphi \text{ or } H, t \models_M \psi, \\
H, t \models \bigcirc\varphi & \text{iff} & H, t+1 \models_M \varphi, \\
H, t \models K_i\varphi & \text{iff} & \text{for all } H' \in \mathcal{H},\ H_t \sim_i H'_t \text{ implies } H', t \models_M \varphi, \\
H, t \models \varphi U \psi & \text{iff} & \text{there exists } k \geq t \text{ such that } H, k \models_M \psi \text{ and,} \\
 & & \text{for all } l,\ t \leq l < k \text{ implies } H, l \models_M \varphi.
\end{array}
$$

Reference

"A Knowledge Based Semantics of Messages", R. Parikh and R. Ramanujam, Journal of Logic, Language and Information, 12(4), pp. 453-67, 2003.

doi.org/10.1023/A:1025007018583

Semantics

The semantics of the preference modality is given as follows.

$$H, t \models \Diamond_i \varphi \quad \text{iff} \quad \exists H'. \, H \preceq_i H' \text{ and } H', t \models \varphi$$

The dual of the $\Diamond_i$ is denoted by $\Box_i$, and defined in the usual sense:

$\Box_i \varphi = \neg \Diamond_i \neg \varphi$.

Axiomatisation

- All tautologies of propositional logic,
- $K_i(\varphi \to \psi) \to (K_i\varphi \to K_i\psi)$,
- $K_i\varphi \to \varphi \wedge K_iK_i\varphi$,
- $\neg K_i\varphi \to K_i\neg K_i\varphi$,
- $\Box_i(\varphi \to \psi) \to (\Box_i\varphi \to \Box_i\psi)$,
- $\Box_i\varphi \to \varphi$,
- $\Box_i\varphi \to \Box_i\Box_i\varphi$,
- $\bigcirc(\varphi \to \psi) \to (\bigcirc\varphi \to \bigcirc\psi)$,
- $\bigcirc\neg\varphi \leftrightarrow \neg \bigcirc \varphi$,
- $\varphi U\psi \leftrightarrow \psi \vee (\varphi \wedge \bigcirc(\varphi U\psi))$.

Rules of Inferences

The rules of inference are modus ponens, and necessitation for all three modalities:

- $\vdash \varphi, \varphi \rightarrow \psi \therefore \vdash \psi$,
- $\vdash \varphi \therefore \vdash K_i\varphi$,
- $\vdash \varphi \therefore \vdash \Box_i\varphi$
- $\vdash \varphi \therefore \vdash \bigcirc\varphi$,
- $\vdash \varphi \rightarrow (\neg\psi \wedge \bigcirc\varphi) \therefore \vdash \varphi \rightarrow \neg(\varphi U \psi)$.

Completeness and Decidability

Theorem

This system with (static) preferences is decidable, sound and complete.

Updating Preferences

Dynamic Modality

We will carry out the preference update by a **distinguishing formula** φ.

The formula φ is a “distinguishing formula” for H, t and H', t, if $H, t \models \varphi$ but $H', t \models \neg\varphi$.

For a given $H \preceq_i H'$, the purpose of a preference update by a distinguishing formula φ is to eliminate $H \preceq_i H'$ from the preference relation so that $H \not\preceq_i H'$ is obtained.

We denote the updated preference relation for agent i by $\preceq_i^*$, and the preference update by φ with $[\varphi!]$.

Model and Syntax

The preference update model $M!\varphi$ with respect to the distinguishing formula φ is a tuple

$$M!\varphi = (\mathsf{E}, \mathcal{H}, \mathsf{A}, \{\mathsf{E}_i\}_{i\in\mathsf{A}}, \{\lambda_i\}_{i\in\mathsf{A}}, \{\preceq_i\}_{i\in\mathsf{A}}, \{\preceq_i^*\}_{i\in\mathsf{A}}, V)$$

where the updated preference orders $\preceq_i^*$ are defined as

$$\preceq_i^* := \preceq_i \setminus \{(H, H') : H, t \models_M \varphi \text{ and } H', t \models_M \neg\varphi \text{ for any } t\}.$$

For a preference order $\preceq_i$ and a formula φ, the updated relation $\preceq_i^*$ with respect to the distinguishing formula φ is also a preference order.

Syntax and Semantics

The language $\mathcal{L}^*$ of this system is specified as follows for $p \in P$, $i \in \mathrm{A}$:

$$\varphi := \; p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid K_i\varphi \mid \bigcirc\varphi \mid \varphi U \varphi \mid \Diamond_i\varphi \mid [!\varphi]\varphi$$

Given a model M and a distinguishing formula φ, the semantics of the preference update modality is given as follows.

$$H, t \models_M [\varphi!]\psi \quad \text{iff} \quad H, t \models_{M!\varphi} \psi$$

We call this system HBPL*.

Additional Axioms and Rules

The additional set of axioms for the dynamic preference modality is given as follows.

- $[\varphi!]p \leftrightarrow p$
- $[\varphi!]\neg\psi \leftrightarrow \neg[\varphi!]\psi$
- $[\varphi!](\psi \wedge \chi) \leftrightarrow [\varphi!]\psi \wedge [\varphi!]\chi$
- $[\varphi!](\psi \vee \chi) \leftrightarrow [\varphi!]\psi \vee [\varphi!]\chi$
- $[\varphi!]K_i\psi \leftrightarrow K_i[\varphi!]\psi$
- $[\varphi!]\bigcirc\psi \leftrightarrow \bigcirc[\varphi!]\psi$
- $[\varphi!](\psi U \chi) \leftrightarrow ([\varphi!]\psi)U([\varphi!]\chi)$
- $[\varphi!]\Diamond_i\psi \leftrightarrow (\neg\varphi \wedge \Diamond_i[\varphi!]\psi) \vee \Diamond_i(\varphi \wedge [\varphi!]\psi)$

The additional proof rule: $\vdash \psi \therefore \vdash [\varphi!]\psi$.

Completeness and Decidability

Theorem

HBPL* is complete with respect to the axiomatization given.

Theorem

HBPL* is decidable.

Completeness and Decidability

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Theorem

HBPL* is decidable.

Completeness proof relies on the following observation.

Reduction Lemma

Every formula in HBPL* can be rewritten as a logically equivalent update-free formula.

Completeness

The Boolean cases for this reduction are immediate.

Let us consider the epistemic case for a given model M.

We start with $H, t \models_M [\varphi!]K_i\psi$.

$$
\begin{array}{lll}
H, t \models_M [\varphi!]K_i\psi & \text{iff} & H, t \models_{M!\varphi} K_i\psi \\
 & \text{iff} & \forall H'.H_t \sim_i H'_t, H', t \models_{M!\varphi} \psi \\
 & \text{iff} & \forall H'.H_t \sim_i H'_t, H', t \models_M [\varphi!]\psi \\
 & \text{iff} & H, t \models_M K_i[\varphi!]\psi
\end{array}
$$

Completeness

The case for the preference modality is interesting.

$H, t \models_M [\varphi!]\Diamond_i\psi$	iff	$H, t \models_{M!\varphi} \Diamond_i\psi$
		Case 1: φ is not satisfied at the current state
	iff	$H, t \models_{M!\varphi} \Diamond_i\psi$ and $H, t \models_M \neg\varphi$
	iff	$H, t \models_M \neg\varphi$ and $\exists H'. H \preceq_i H'$ such that
		$H', t \models_{M!\varphi} \psi$
	iff	$H, t \models_M \neg\varphi$ and $H', t \models_M [\varphi!]\psi for H \preceq_i H'$
	iff	$H, t \models_M \neg\varphi$ and $H, t \models_M \Diamond_i[\varphi!]\psi$
	iff	$H, t \models_M \neg\varphi \wedge \Diamond_i[\varphi!]\psi$

Completeness

The case for the preference modality is interesting.

		Case 2: $\neg\varphi$ isn't satisfied at accessible histories
$H, t \models_M [\varphi!]\Diamond_i\psi$	iff	$H', t \models_{M!\varphi} \psi$ for $H \preceq_i H'$
		(as H' cannot satisfy $\neg\varphi$ in M)
	iff	$H', t \models_{M!\varphi} \psi$ for $H \preceq_i H'$ and $H', t \models_M \varphi$
	iff	$H', t \models_M [\varphi!]\psi$ and $H', t \models_M \varphi$ for $H \preceq_i H'$
	iff	$H', t \models_M [\varphi!]\psi \wedge \varphi$ for $H \preceq_i H'$
	iff	$H, t \models_M \Diamond_i(\varphi \wedge [\varphi!]\psi)$
		(combining *Cases 1* and *2* disjunctively:)
	iff	$H, t \models_M (\neg\varphi \wedge \Diamond_i[\varphi!]\psi) \vee \Diamond_i(\varphi \wedge [\varphi!]\psi)$

Some Applications

Relevance for Other Logics: Arrow Updates

An alternative semantics for modal dynamics is to eliminate the relation. This method is called “arrow updates”.

We showed that HBPL* models can be transformed into Arrow Update models.

Arrow Updates

“Arrow Update Logic”, B. Kooi and B. Renne, The Review of Symbolic Logic, 4(4), pp. 536-559, 2011.

doi.org/10.1017/S1755020311000189

Relevance for Other Logics: Product Updates

An interesting strategy to incorporate Kripke models into history based models is to combine histories/events with states or possible worlds. This approach generates a cartesian product of history-time pairs and states, producing a complex and expressive system.

If histories are thought of expressing sequences of events taking place over *time*, then states can be thought of describing them over *space*. Their cartesian product, therefore, describes how histories develop over time and space. This provides histories with extensionality.

We showed that HBPL* can be integrated into product updates and the resulting system is complete.

Next

A next step is to apply these ideas to define game equilibria/solutions.

A long-term goal is to use this system for program runs and verification as well as certain issues in AI ethics such as *the trolley problem*.

References

Reference

Reference

"A History Based Logic for Dynamic Preference Updates", Can Başkent and Guy McCusker, Journal of Logic Language and Information, 2019.

doi.org/10.1007/s10849-019-09307-1

Slogan!

Rational agents

update

their preferences and strategies!

Thank you!

Talk slides and the papers are available at my website

CanBaskent.net/Logic