

# Logic in an Illogical World

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From ideal reasoners to  
real-life reasoners,  
logic has a lot to say!

# What is Logic?

**Logic is the study of reasoning.**

However, it uses a normalised set up.

Reasoners are often ideal agents with almost infinite capacity to reason.

# What is Logic?

**Logic is the study of reasoning.**

They can know infinitely-many facts, they can reason infinitely-many times. And they are perfectly rational.

This is a side-effect of mathematical reasoning.

The more mathematical logic has got, more *unrealistic* and even *illogical* it became.

# What is Logic?

This is how logic of today approach a simple problem. Imagine the following situation:

Alice and Bob are enjoying some tea in a café in London. Suddenly, the rain starts, and they both realise it. Each of them also realises that the other person realised it, too. Rain becomes common knowledge between Alice and Bob.

# What is Logic?

We formalise this simple situation in logic as follows:

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This is an infinitary formula that we need to express that the rain has become common knowledge between Alice and Bob.

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This is too much.

The simple situation turned into a mathematically sound, but infinitary formula, where agents can (and must) reason infinitely many times.

# Why Logic?

Logic is an old discipline.

It has had a long impact.

Since antiquity, it has been getting more and more mathematical and symbolic.



Aristotle

# Why Logic?

“For if A is predicated of every B, and B of every C, A must necessarily be predicated of every C, for it has been before shown, how we predicate “of every;” so also if A is predicated of no B, but B is predicated of every C, A will not be predicated of any C.”

*Prior Analytics*, Aristotle



Aristotle

# Today: Even More Mathematical

This is how logic papers look now.

## A History Based Logic for Dynamic Preference Updates

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- iff  $H, t \models_{M! \varphi} \Diamond_i \psi$  and  $H, t \models_M \neg \varphi$
- iff  $H, t \models_M \neg \varphi$  and  $\exists H'. H \preceq_i H'$  such that  
 $H', t \models_{M! \varphi} \psi$
- iff  $H, t \models_M \neg \varphi$  and  $H', t \models_M [\varphi!] \psi$  for  $H \preceq_i H'$
- iff  $H, t \models_M \neg \varphi$  and  $H, t \models_M \Diamond_i [\varphi!] \psi$
- iff  $H, t \models_M \neg \varphi \wedge \Diamond_i [\varphi!] \psi$   
(Case 2 :  $\neg \varphi$  is not satisfied at accessible histories)
- iff  $H', t \models_{M! \varphi} \psi$  for  $H \preceq_i H'$   
(as  $H'$  cannot satisfy  $\neg \varphi$  in  $M$ )
- iff  $H', t \models_{M! \varphi} \psi$  for  $H \preceq_i H'$  and  $H', t \models_M \varphi$
- iff  $H', t \models_M [\varphi!] \psi$  and  $H', t \models_M \varphi$  for  $H \preceq_i H'$
- iff  $H', t \models_M [\varphi!] \psi \wedge \varphi$  for  $H \preceq_i H'$
- iff  $H, t \models_M \Diamond_i (\varphi \wedge [\varphi!] \psi)$

# Today: Even More Mathematical

Logic evolved into

- theoretical computer science and algorithms
- set theory
- cognitive science
- animal/children's reasoning
- truth theory and paradoxes
- economics and game theory

and many more...

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# Why Economics?

Historically, logic's role in computational, mathematical and philosophical areas was natural and a bit expected.

If you study reasoning, you can study how computers could and should reason, and what kind of mathematics is behind this. You can also study the truth behind reasoning.

But, economics?

# Why Economics?

Some fields in economics study the way *homo economicus* reasons. Markets, buyers, sellers, negotiators, bargainers, customers, corporations all are part of this.

Auctions, stock markets, bargaining etc are daily-life cases where the way *homo economicus* reasons determines the outcome.

But, that is logic.

# Difference in Economics?

But, economics is a social science. It deals with people and organisations.

Economical agents, *homo economicus*, are not ideal agents: they don't have endless memory or infinitary reasoning power.

# Difference in Economics?

Because of that, logic evolved into a discipline of studying how humans reason under economical parameters.

For example, how they choose between ice-cream and broccoli, stock market and savings etc.

“What if you are single, and hope to appeal as many attractive potential dating partners as possible at an upcoming singles event? My advice would be to bring a friend who has your basic physical characteristics (similar coloring, body type, facial features) but slightly less attractive(-you).”

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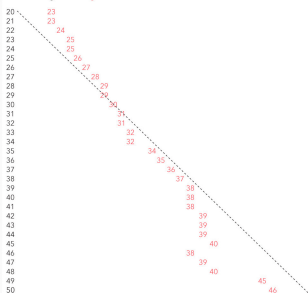
People’s preferences thus are not rational. The way they reason is not logical in the classical, Aristotelian sense.

**Dan Ariely**

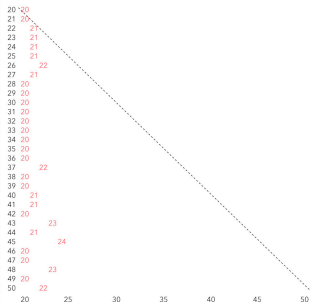
Predictably Irrational, Dan Ariely, Harper, 2008.

# Logic and Economics

a woman's age vs. the age of the men who look best to her



a man's age vs. the age of the women who look best to him



Christian Rudder

Dataclysm: Who We Are (When We Think No One's Looking),  
Christian Rudder, Crown Publishers, 2014.

Thus, people are irrational and illogical.

The way the people reason, and the way that logic dictates are so different from each other.

Similarly, the way the social groups (companies, governments, masses) reason is even more different than all the others.



Then, the question is whether mathematical methodology of logic is strong enough to explain the irrational and the illogical.

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**Can logic explain the illogical? Can logic work with the illogical?**

# From Classical to Non-Classical Logic

From physics to biology, we have seen many non-classical and improved theories replacing the classical and old ones.

Logic is not an exception.

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We have non-classical theories of logic as well.

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**Fuzzy Logic** talks argues there are infinitely-many different values between true and false.

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The hope is that it will explain more, understand better and predict more accurately.

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“Machine” includes AI and cars.

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**What is next?**

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real-life reasoners,  
logic has a lot to say!

# Thank you!

Talk slides are available at my website

[CanBaskent.net/Logic](http://CanBaskent.net/Logic)