Inconsistent Inquiries

Learning from and Working with Contradictions in Heuristics

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Today's Plan

1. Introduction
2. Formalising Inquiry
3. Ideas for Future Work

Slogan!

Inconsistencies promote

knowledge growth.

We need to know how to *work with* them!

Introduction

Motivation

Contradictions (or *monsters*) are an essential part of Lakatosian heuristics. Depending on their characteristics, the theory is revised.

Similarly, Hintikka “brackets” abnormalities in order to obtain a classical, contradiction-free system for epistemic inquiries.

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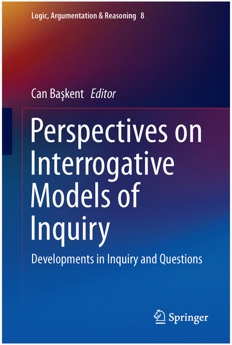

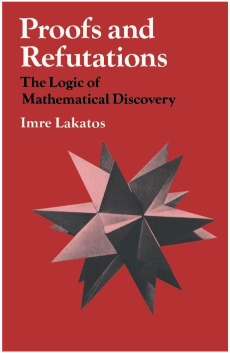

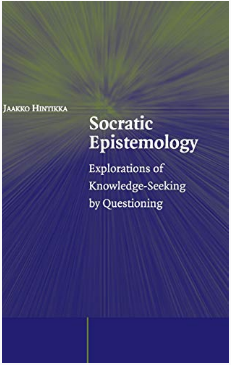

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Challenge

How can we represent this **process** using logic?

Examples: Lakatosian Methodology

1. Primitive conjecture.
2. Proof (a rough thought experiment or argument, decomposing the primitive conjecture into subconjectures and lemmas).
3. Global counterexamples.
4. Proof re-examined. The guilty lemma is spotted. The guilty lemma may have previously been hidden or misidentified.
5. Proofs of the other theorems are examined to see if the newly found lemma occurs in them.
6. Hitherto accepted consequences of the original and now refuted conjecture are checked.
7. Counterexamples are turned into new examples, and new fields of inquiry open up

D. Corfield, “Assaying Lakatos’s History and Philosophy of Science”, *Studies in History and Philosophy of Science*, vol. 28, pp. 99-121, 1997.

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As such this system is *paraconsistent*: it is a logical system that does not explode under the existence of contradictions.

Examples: Lakatosian Methodology

Lakatosian method of proofs and refutations has many non-classical (logical) elements:

- "Proofs that do not prove"
- "Proof attempts"
- Role of counter-examples
- Monster-barring

Examples: Lakatosian Methodology

The battle between rigour and axiomatisation is an entertaining passage in *Proofs and Refutations*:

> *The Cauchy revolution of rigour was motivated by a conscious attempt to apply Euclidean methodology to the Calculus. He and his followers thought that this was how they could introduce light to dispel the 'tremendous obscurity of analysis'. Cauchy proceeded in the spirit of Pascal's rules: he first set out to define the obscure terms of analysis –like limit, convergence, continuity etc– in the perfectly familiar terms of arithmetic, and then he went on to prove everything that had not previously been proved, or that was not perfectly obvious.* ***Now in the Euclidean framework there is no point trying to prove what is false*** *(My emphasis), so Cauchy had first to improve the extant body of mathematical conjectures by jettisoning the false rubbish.*

Examples: Lakatosian Methodology

> *(...) What was considered by the rigourists to be hopeless rubbish, such as conjectures about sums of divergent series, was duly committed to the flames. 'Divergent series are' wrote Abel, 'the work of the devil'. They only cause 'calamities and paradoxicalities'. (...) The idea of a proof which deserves its name and still is not conclusive was alien to the rigourists.*

▷ Note the *Hume*ist reference!

Lakatos, *Proofs and Refutations*, p. 137 (footnotes are omitted), Cambridge University Press, 2007.

Example: Hintikkan Inquisitive Inquiry

Hintikka's theory of interrogative models of inquiry is a well-known example of a dynamic epistemic procedure that results in knowledge increase.

In an interrogative inquiry, the inquirer is given a theory and a question. He then tries to answer the question based on the theory by posing some questions to nature or an oracle. In an interrogative inquiry, the inquirer has two options. He is allowed to ask questions to nature/oracle, conceived as a truthful source of information, or alternatively draw conclusions by using the given base theory and the answers he has already received.

Example: Hintikkan Inquisitive Inquiry

Hintikka's *Inquisitive Models of Inquiry* present a rather non-classical methodology.

> *An important aspect of this general applicability of the interrogative model is its ability to handle uncertain answers – that is, answers that may be false. The model can be extended to this case simply by allowing the inquirer to tentatively disregard ("bracket") answers that are dubious. (...) Equally obviously, further inquiry might lead the inquirer to reinstate ("unbracket") a previously bracketed answer. This means thinking of interrogative inquiry as a self-corrective process."*

Hintikka, *Socratic Epistemology*, Cambridge University Press, 2007.

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This is a paraconsistent reasoning.

Example: Hintikkan Inquisitive Inquiry

In an inquiry or a dialogue game, how can we know which answers to ignore beforehand? How can we know what to reject or accept?

A dynamic and empirical method can be proposed:

We simply choose the assumptions and responses that help us win the game. If we can win the game with a particular set of assumptions, then we adopt these assumptions as they give us a win. If we fail to win those assumptions and the answers we received in the inquiry, we simply select another set of assumptions and answers, and keep playing, and repeat the procedure if necessary.

Putting It All Together

There are some common elements in Lakatosian and Hinttikan thought that relate to paraconsistency and dialetheia.

- Both Lakatosian and Hintikkan methods are about knowledge increase caused by empirical testing,
- When the empirical test produces a contradictory result, both Lakatosian and Hintikkan methods have some constructive strategy to follow,
- Both Lakatosian and Hintikkan methods have some erotetic aspects,
- Both Lakatosian and Hintikkan methods are seen as *activities*.

Thus, they are processes, they are dynamic.

And they are game-like.

Formalising Inquiry

Inquiry as a Game

I consider inquiry as a question-answer game with a hint of paraconsistency. This is how I formalise ParaIMI.

Let Σ be a set of moves common to set of players n. We denote the set of propositional variables by P.

We interpret the game on a tree where each node denotes a game state, and the labeled edges indicate the moves.

The Logic PARAIMI

A PARAIMI game tree T is defined as a tuple $T = (W, \Rightarrow, w_0)$ where W is a non-empty set with $w_0 \in W$. The partial function $\Rightarrow: W \times \Sigma \mapsto W$ specifies the labeled edges of the tree where the labels represent the moves.

The extensive form PARAIMI game tree then is the pair (T, t) where T is as before and $t : W \mapsto n$ specifies whose turn it is at each state.

A PARAIMI model M then is defined as the tuple $M = (T, t, V)$ where T and t are as before, and V is a valuation function assigning subsets of W to $\mathbf{P}$.

A *strategy* is defined as a subtree of T that specifies what moves to play for a certain player when it is his turn.

Strategy s_i for a player $i \in n$ is a function $s_i : W_i \mapsto \wp(\Sigma)$ where W_i is the set of states where it is i's turn to play (those $w \in W$ where $t(w) = i$).

The Logic ParaIMI

Strategies give rise to strategy trees per player, and the formulas are evaluated at strategy trees.

Because, an inquiry is a strategic process, not an automated, fully deductive reasoning. Second, strategy trees localise the model. This is an important point from a game theoretical perspective. The agents have local information and local strategies, as they may not know their opponent's strategy. Finally, this formalism allocates the oracles. The information that the oracle provides can be taken as a precondition when it is evaluated at a strategy tree.

The Logic ParaIMI

For a player i, and a strategy s_i, the strategy tree $T_{s_i} = (W_{s_i}, \Rightarrow_{s_i}, w_0, t_{s_i})$ is the least subtree of T satisfying

1. $w_0 \in W_{s_i}$,
2. for any $w \in W_{s_i}$, if $t(w) = i$; then there exists a $w' \in W_{s_i}$ and a move a such that $w \overset{a}{\Rightarrow}_{s_i} w'$.
   On the other hand, if $t(w) \neq i$, then for all w' with $w \overset{a}{\Rightarrow} w'$ for some a, we have $w \overset{a}{\Rightarrow}_{s_i} w'$.

The Logic PARAIMI

The syntax of PARAIMI is given as follows.

$$p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \rightsquigarrow_i a$$

where $p \in \mathsf{P}$ and $a \in \Sigma$.

The formula $\varphi \rightsquigarrow_i a$ means that when the precondition φ is satisfied, then the agent i makes an a move. Ii is a controller to see whether the move a is compatible with the strategy or not.

This is why we evaluate formulas at the strategy trees.

The Logic ParaIMI

In a scientific inquiry for instance, the formula $\varphi \rightsquigarrow_i a$ can denote the steps to take after making an observation.

When scientist i makes an observation φ, then based on this observation, he proceeds to some action a (some further experimentation, calculation, revision, etc).

The key point is that the whole process $\varphi \rightsquigarrow_i a$ may or may not agree with his strategy, which is his research program aiming at proving a scientific hypothesis in this case.

Therefore, we need a semantics for this formula $\varphi \rightsquigarrow_i a$.

The Logic ParaIMI

In ParaIMI, the semantics of the Booleans are standard. Let us discuss the dynamic operator.

$$T_{s_i}, w \models \varphi \rightsquigarrow_i a \qquad \text{iff} \qquad M, w \models \varphi \text{ implies } a = out_s(w)$$

where $out_s(w)$ is an outgoing labeled edge from w with respect to the strategy s.

The semantical definition simply indicates that when a precondition is met, the strategy tells us which action to take.

The Logic ParaIMI

Let us give the truth table for the paraconsistent ParaIMI, which relies on Priest's *Logic of Paradox*.

	$\neg$
T	F
P	P
F	T

$\wedge$	T	P	F
T	T	P	F
P	P	P	F
F	F	F	F

$\vee$	T	P	F
T	T	T	T
P	T	P	P
F	T	P	F

where P stands for the paradoxical/paraconsistent truth value.

The Logic ParaIMI

This logic is paraconsistent because $\varphi, \neg\varphi \models \psi$ does not always hold.

Similarly, the resolution and modus ponens do not hold either in Logic of Paradox:

- $\varphi, \neg\varphi \vee \psi \models \psi$
- $\varphi, \varphi \to \psi \models \psi$

Graham Priest, "The Logic of Paradox", *Journal of Philosophical Logic*, vol. 8, pp. 219-241, 1979.

The Logic PARAIMI: Some Results

Let us first observe how the *oracle* works.

Theorem

In PARAIMI, the subtree that is constructed after the responses of the oracle is a subtree of the strategy tree of the Inquirer.

Theorem

An inquiry that is conducted based on the oracle's responses at each state is the unique minimal strategy tree.

Theorem

Every strategy has a unique minimal strategy that coincides with the oracle's strategy.

The Logic ParaIMI: Some Results

Theorem

The best strategy for the inquiry game in ParaIMI can be obtained by introducing the oracle's answers and the assumption of state-based rationality as a set of premise to the inquiry.

Theorem

The computational complexity of asking questions to the oracle and constructing a rational strategy tree in ParaIMI is the same as performing the same operation in the classical IMI.

Theorem

Interrogative inquiries conducted with an oracle with state-based rationality are determined.

Applications of ParaIMI

It is not difficult to apply ParaIMI to Lakatosian and Hintikkan methodologies, including *monster-barring* and *inquisitive inquiries*.

CB, “Towards Paraconsistent Inquiry”, *The Australasian Journal of Logic*, vol. 13, no. 2, pp. 21-40, 2016.

Ideas for Future Work

Inspiration for Future Work

- Paraconsistent logic with inquisitive semantics,
- Game semantics for interrogation,
- Oracles in game semantics [partially done],
- ParaIMI and learning theory,
- AI and interrogative models of inquiry.

Conclusion

In this talk, I attempted at unearthing the hidden connections between the Hintikkan and Lakatosian methodologies via paraconsistency.

Being classical logicians, neither paid close enough attention to paraconsistency. Yet, I argued that non-classical elements in both systems are transparent and *instrumental* for their methodologies. Thus, it is a valuable attempt to formalise them.

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Slogan!

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Thank you!

Talk slides and the papers are available at my website:

canbaskent.net/logic