Game Semantics for Logics of Nonsense

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Today's Plan

1. Motivation
2. Introduction to Game Semantics
3. Logics of Nonsense
4. Game Semantics for Logics of Nonsense
5. Extending Nonsense Games
6. Conclusion

Slogan!

Non-classical logics help us understand

the connection between logic and games

in a more nuanced way!

Motivation

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Once an error/bug appears in a program, it may propagate throughout the program. It dominates the output.

The expression "$1/x$" is considered meaningless when $x = 0$.

Certain approaches to truth disregard paradoxes of self-reference and exclude them as *meaningless*.

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Challenge

How can we explain this phenomenon using game theoretical semantic tools?

Introduction to Game Semantics

Why Game Semantics?

Game theoretical semantic tools offer a very intuitive and natural approach to semantics.

They suggest *computational connections* between truth, proofs, programs and strategies, relating major concepts of game theory, computer science and logic to each other constructively.

Game semantics is perhaps the most studied *non-compositional* semantics. It is non-compositional in the way that the truth of a complex formula is evaluated based on the truth values of *some* of its components.

Game theoretical analysis of “infectiousness” helps us draw a broader picture of *interactive* and rational behaviour, which is a central theme in multi-agent systems, social choice and decision theories.

The Game, classically

In a semantic game, the given formula is broken into subformulas step by step. The game terminates when it reaches the propositional atoms. If the game ends with a true atom, then the verifier wins the game. Otherwise, it is a win for the falsifier.

The moves and turns of the game are determined syntactically based on the shape of the formula. If the main connective is a conjunction, the falsifier makes a move. If it is disjunction, the verifier makes a move. If the main connective is a negation, the players switch roles: the verifier becomes the falsifier, the falsifier becomes the verifier.

The Game, strategies

A player has a *winning strategy* if he has a set of rules that guides him throughout the play and tells him which move to make, and consequently gives him a win regardless of how the opponent plays.

In *classical* game semantics, winning strategies necessarily determine the truth values of the formulas.

In non-classical game semantics, this assumption is rejected. Because some games may have multiple winners – with multiple "winning" strategies.

In that case, we need to be able to identify the winning strategy that necessarily determines the truth value of the formula in question. We call such winning strategies *dominant*.

The Game, models

Model

A model M is a tuple (S, v) where S is a non-empty domain on which the game is played, and the valuation function v assigns the formulas in $\mathcal{L}$ to truth values in the logic.

Game tuple

A semantic game is a tuple $\Gamma = (\pi, \rho, \sigma, \tau, \delta)$ where π is the set of players, aiming at winning the game by reaching atomic formulas with specific truth values based on their roles, ρ is the set of well-defined game rules, σ is the set of positions, τ is the set of positions of the game-token in the case of a concurrent play, and δ is the set of designated truth values.

The Game, models

Strategies

In a semantic game, a *dominant winning strategy* is a winning strategy that determines the truth value of φ once played.

Logics of Nonsense

The Logic

Bochvar–Halldén Logic introduces an additional truth value N, called *nonsense*, which intuitively stands for sentences which are nonsensical or meaningless.

Let $\mathcal{L}$ be the propositional language.

The truth table is given as follows.

We call this logic BH3.

	$\neg$
T	F
N	N
F	T

$\wedge$	T	N	F
T	T	N	F
N	N	N	N
F	F	N	F

$\vee$	T	N	F
T	T	N	T
N	N	N	N
F	T	N	F

Game Semantics for Logics of Nonsense

The Game Setup

In a semantic game for BH3, we need three players to force the three truth values. In addition to the classical players Verifier and Falsifier, we introduce a third player which we call “Dominator”.

Dominator forces the game to a nonsense proposition. As such, he is allowed to make moves along other players.

We also stipulate that Dominator’s strategy is *dominant* – his wins determine the truth value.

Rules

Dominator is allowed to make moves along other players.

We stipulate that Dominator's strategy dominates the others and his role does not change throughout the game, even under negation.

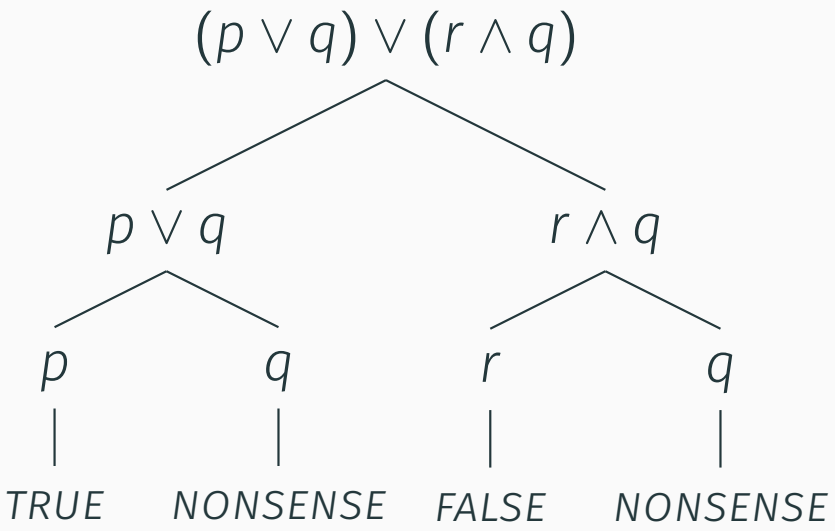
The strategies of Verifier and Falsifier, however, do not dominate each other or any other strategy, by default.

Rules

- For negation, Verifier and Falsifier switch roles. Dominator keeps his role.
- For conjunction, Falsifier and Dominator make a choice independently and simultaneously,
- For disjunction, Verifier and Dominator make a choice independently and simultaneously.

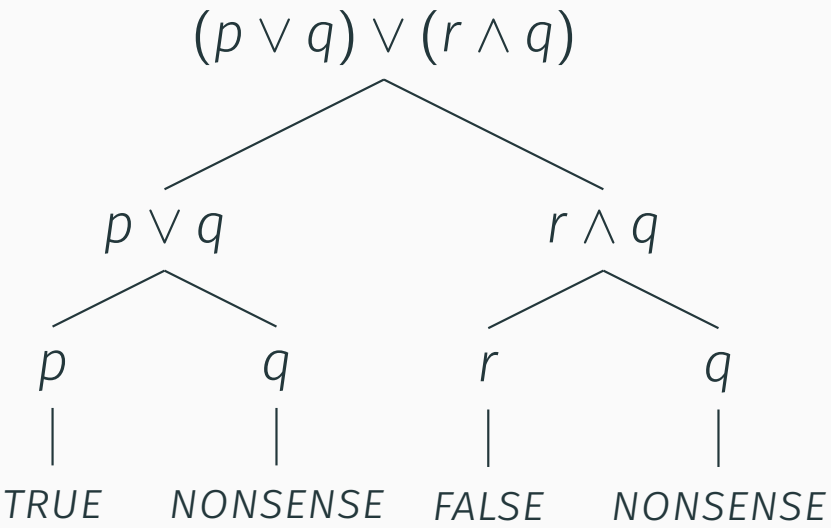
Examples

Consider the formula $(p \vee q) \vee (r \wedge q)$ where p is true, q is nonsense and r is false. This formula has the truth value N in BH3. The following diagram depicts the game tree informally.



In this game, some of the positions in σ are $(\mathrm{Verifier}, (p \vee q) \vee (r \wedge q))$, $(\mathrm{Dominator}, (p \vee q) \vee (r \wedge q))$, $(\mathrm{Verifier}, p \vee q)$, $(\mathrm{Dominator}, p \vee q)$, $(\mathrm{Falsifier}, r \wedge q)$, $(\mathrm{Dominator}, r \wedge q)$.

Examples



First, Verifier and Dominator make choices. at the same time. If Verifier chooses $p \vee q$, then he gets to make the next move and chooses p to win the game. So, he has a *winning* strategy. However, Verifier's winning strategy is dominated.

Simultaneously, Dominator also gets to make a move. Suppose, he chooses $p \vee q$ as well. Now, he can make a move again and chooses q which is nonsense. This constitutes his winning strategy. But, by game rules, his strategy dominates the others.

Results

Theorem

In a semantic game for BH3, Verifier and Falsifier can never have winning strategies at the same time.

Theorem

In a semantic game for BH3,

- Verifier has a dominant winning strategy if and only if φ is true in M,
- Falsifier has a dominant winning strategy if and only if φ is false in M,
- Dominator has a dominant winning strategy if and only if φ is nonsense in M.

Strategy Elimination

Iterated elimination of strictly dominated strategies suggests that by eliminating those strategies that are dominated, we can reach a solution.

This method directly applies to semantic games for BH3. Because

- In BH3 games, the strategies for Dominator strictly dominates the strategies of Verifier and Falsifier.
- In BHS games, Dominator makes a move at each connective.
- An action of a player in a finite strategic game is never a best-response if and only if it is strictly dominated.

From Strategies to Truth

This observations allow us see the correctness theorem in a different way.

Theorem

In a BH3 semantic game for φ, if φ contains a literal with a truth value nonsense, then Dominator has a dominant winning strategy and consequently φ is nonsense.

Extending Nonsense Games

From Three to Four Values

Using the idea of dominant strategies, we can *engineer* some semantic games without first considering their possible semantics. This is an interesting methodology which develops logics (and truth tables) which are *solely* generated by semantic games.

Let us introduce a fourth player, called *Dictator*. Dominator's strategy dominates Falsifier's and Verifier's, whereas Dictator's strategy dominates them all.

For simplicity, we call the truth value that is forced by Dictator as *super* and denote it by S. Thus, Dictator's role is to force the semantic game to an end with an atom with the truth value super.

A Logic for Four Players

This game generates the following truth-table and logic.

	$\neg$
T	F
F	T
N	N
S	S

$\wedge$	T	N	S	F
T	T	N	S	F
N	N	N	S	N
S	S	S	S	S
F	F	N	S	F

$\vee$	T	N	S	F
T	T	N	S	T
N	N	N	S	N
S	S	S	S	S
F	T	N	S	F

Rules for Four Players

- If φ is atomic, the game terminates, and Verifier wins if φ is true, Falsifier wins if φ is false, Dominator wins if φ is nonsense, and Dictator wins if φ is super,
- if $\varphi = \neg\psi$, Falsifier and Verifier switch roles, Dominator and Dictator keep their roles, and the game continues as $\Gamma(M, \psi)$,
- if $\varphi = \chi \wedge \psi$, Falsifier, Dominator and Dictator choose between χ and ψ simultaneously,
- if $\varphi = \chi \vee \psi$, Verifier, Dominator and Dictator choose between χ and ψ simultaneously.
- Dominator's strategy strictly dominates Verifier's and Falsifier's, and Dictator's strategy strictly dominates them all.

Results

Theorem

In a BH4 semantic games,

- Verifier has a dominant winning strategy if and only if φ is true in M,
- Falsifier has a dominant winning strategy if and only if φ is false in M,
- Dominator has a dominant winning strategy if and only if φ is nonsense in M,
- Dictator has a dominant winning strategy if and only if φ is super in M,

Beyond Four Players

Following our methodology, different combinations of the above game rules can be constructed, yielding different logics with more or different infectious truth values.

Moreover, additional truth values, thus players, beyond the fourth can be introduced in a way that the dominant winning strategies form a linear order: Dominator dominates the classical players; Dictator dominates Dominator; a fifth player, say King, dominates Dictator and the rest etc.

This procedure is rather straight-forward for countably many players. The semantic games seem to get more interesting once $>\omega$-many players are considered with a linear or branching order of dominant strategies.

Beyond Logics of Nonsense

The idea of using strategy dominance in semantic games for many-valued, non-classical logics is a prolific idea.

This methodology applies directly to Priest's Logic of Paradox and Strong Kleene logic, to name a few.

Conclusion

Conclusion

We showed how non-classical game theory helps us understand the nuances of non-classical logics as well as certain game theoretical concepts.

To the best of our knowledge, this is the first game theoretical semantics suggested for Bochvar–Halldén logics and infectious logics in general, relating infectiousness to strategy dominance.

Next, we can relate strategy dominance in semantic games to computing game equilibria and its complexity.

Furthermore, studying a branching order of strategy dominance and how they relate to truth-coalitions in game semantics remain an interesting direction.

Some References

Reference

Can Başkent, “Game Theoretical Semantics for Some Non-Classical Logics”, *Journal of Applied Non-Classical Logics*, vol.26(3), pp. 208-39, 2016.

Reference

Can Başkent & Pedro Henrique Carrasqueira, “A Game Theoretical Semantics for a Logic of Formal Inconsistency”, *Logic Journal of the IGPL*, 2018.

Reference

Dimitri Bochvar, “On a Three-valued Logical Calculus and Its Application to the Analysis of the Paradoxes of the Classical Extended Functional Calculus”, *History and Philosophy of Logic*, vol.2(1-2), pp. 87-112, 1981.

Reference

Reference

Thomas M. Ferguson, “Logics of Nonsense and Parry Systems”, *Journal of Philosophical Logic*, vol. 44(1), pp. 65-80, 2015.

Reference

Damien Enrique Szmuc, “Defining LFIs and LFUs in Extensions of Infectious Logics”, *Journal of Applied Non-Classical Logics*, vol. 26(4), pp. 286-314, 2016.

Slogan!

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Thank you!

Talk slides and the papers are available at my website

CanBaskent.net/Logic